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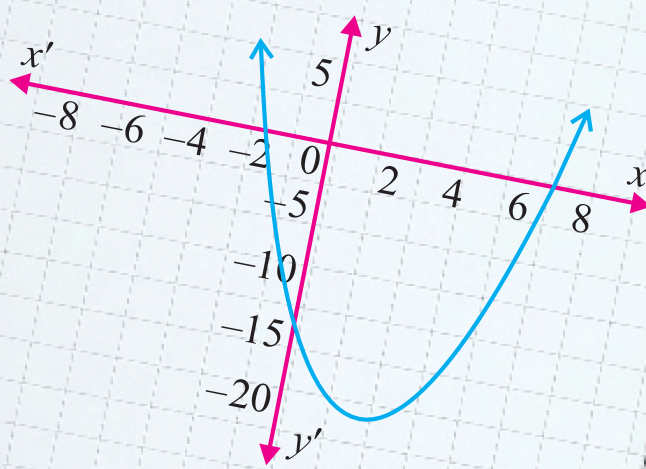
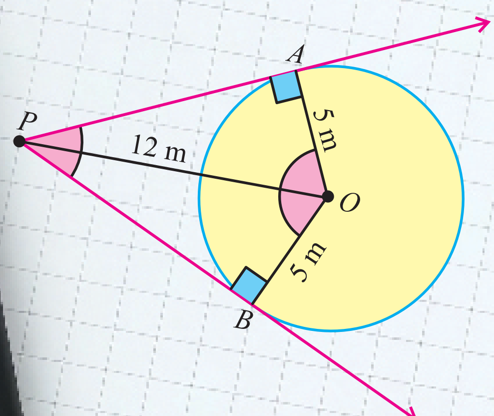
UNIQUE NOTES

MATHEMATICS 10

According to the New Curriculum of PECTAA (2026-27)





$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

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Letter from the Research & Development (R&D) Department

Dear Respected Educators,

- It gives us great pleasure to present the sample chapter of our newly developed notes for the academic session 2026–27.
- These notes have been prepared with dedication and careful planning by our Research & Development team in line with the latest curriculum requirements.
- Designed to meet modern educational standards and student needs, these notes include accurate textbook solutions, additional questions, exam-focused practice material, and clear explanations to support excellent results.
- We believe that quality guidance and smart preparation lead to student success. Therefore, these notes aim to help students excel academically and compete for top positions.
- We are pleased to share this first chapter so your institution may begin planning and preparation while the complete books are being finalized.
- We sincerely hope these notes will prove valuable for your teachers and students. Your trust continues to inspire us to maintain the highest standards of educational excellence.

With best regards,

Ziyad Khan
Principal
Research & Development Department

CONTENTS

Unit 01 COMPLEX NUMBERS

1. Complex Numbers
2. Algebraic Operations on Complex Numbers
3. Complex or Argand Plane
4. Modulus of a Complex Number
5. Finding Real and Imaginary Parts of Complex Number
6. Solution of Simultaneous Linear Equations with Complex Coefficients

Q. Who discovered the complex numbers and when? **10301001**

Ans. The idea of complex numbers first emerged in 16th century, when Italian mathematician Gerolamo Cardano discovered that equations could still be solved when they involved the square root of negative number. In the 18th century, Carl Friedrich Gauss (German mathematician and astronomer) expand this early idea.

Q. Define a complex number. **10301002**

Ans. Complex Number

A complex number is a number expressed in the form $x+iy$, where x and y are real numbers and i is the imaginary unit, defined by $i=\sqrt{-1}$ and $i^2=-1$.

Example 1: Simplify the following:

(i) i^7 **(ii)** i^8 **(iii)** i^{17} **(iv)** i^{-25}

Solution:

(i) $i^7 = i^6 \times i$ **10301003**

$= \left(i^2\right)^3 \times i = (-1)^3 \times i = -1 \times i = -i$

(ii) $i^8 = \left(i^2\right)^4$ **10301004**

$= (-1)^4 = 1$

(iii) $i^{17} = i^{16} \times i$ **10301005**

$= \left(i^2\right)^8 \times i = (-1)^8 \times i = 1 \times i = i$

(iv) $i^{-25} = \dfrac{1}{i^{25}} = \dfrac{1}{i^{24} \times i}$ **10301006**

$= \dfrac{1}{\left(i^2\right)^{12} \times i} = \dfrac{1}{(-1)^{12} \times i} = \dfrac{1}{1 \times i}$

$= \dfrac{1}{i} = \dfrac{1}{i} \times \dfrac{i}{i} = \dfrac{i}{i^2} = \dfrac{i}{-1} = -i$

Q. What are the real and imaginary parts of a complex number? **10301007**

Ans. A complex number is of the form $x+iy\,(or\ x+yi)$, where x and y are real numbers, x is called the real part and y is called the imaginary part of the complex number.

It is customary to denote the standard rectangular form of a complex number $x+iy$ as z and we write $x=\operatorname{Re}(z)$ and $y=\operatorname{Im}(z)$.

For example, $\operatorname{Re}(5-7i)=5$ and $\operatorname{Im}(5-7i)=-7$.

Q. Under what conditions, a complex number is pure imaginary and real? **10301008**

Ans. A complex number is of the form $x+iy$

(i) If $x=0$, the complex number is said to be pure imaginary.

(ii) If $y=0$, the complex number is said to be real.

Q. Discuss the equality of two complex numbers. **10301009**

Ans. Equality of Complex Numbers

Two complex numbers $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ are said to be equal if and only if $\operatorname{Re}(z_1)=\operatorname{Re}(z_2)$ and $\operatorname{Im}(z_1)=\operatorname{Im}(z_2)$ i.e. $x_1 = x_2$ and $y_1 = y_2$.

For example, if $\alpha + i\beta = -2+5i$, then $\alpha=-2, \beta=5$.

Example 2: **If $(2x-1)+(y+4)i=5+7i$, then find the values of x and y.** **10301010**

Solution: Given that

$(2x-1)+(y+4)i=5+7i$

Since the two complex numbers are equal, their real and imaginary parts must be equal.

$$2x-1=5 \quad 2x=5+1 \quad 2x=6 \quad x=\frac{6}{2} \quad x=3$$

$$y+4=7 \quad y=7-4 \quad y=3$$

EXERCISE 1.1

1. Simplify the following:

(i) i^5 **10301011**

Solution:

$$i^5=i^4\times i=\left(i^2\right)^2\times i=(-1)^2\times i \qquad \because i^2=-1$$
$$=1\times i=i$$

(ii) i^{16} **10301012**

Solution:

$$i^{16}=\left(i^2\right)^8=(-1)^8 \qquad \because i^2=-1$$
$$=1$$

(iii) $(-i)^{-19}$ **10301013**

Solution:

$$(-i)^{-19}=\frac{1}{(-i)^{19}}$$
$$=\frac{1}{-i^{19}}=\frac{1}{-i\times i^{18}}$$
$$=\frac{1}{i^{18}\times(-i)}=\frac{1}{\left(i^2\right)^9\times(-i)}$$
$$=\frac{1}{(-1)^9\times(-i)}=\frac{1}{(-1)\times(-i)} \qquad \because i^2=-1$$
$$=\frac{1}{i}=\frac{1}{i}\times\frac{i}{i}=\frac{i}{i^2}=\frac{i}{-1}=-i$$

(iv) $27i^{-26}$ **10301014**

Solution:

$$27i^{-26}$$
$$=\frac{27}{i^{26}}=\frac{27}{\left(i^2\right)^{13}}$$
$$=\frac{27}{(-1)^{13}}=\frac{27}{-1} \qquad \because i^2=-1$$
$$=-27$$

(v) $i^{11}+i^5$ **10301015**

Solution:

$$i^{11}+i^5$$
$$=i^{10}\times i+i^4\times i$$
$$=\left(i^2\right)^5\times i+\left(i^2\right)^2\times i$$
$$=(-1)^5\times i+(-1)^2\times i \qquad \because i^2=-1$$
$$=(-1)\times i+(1)\times i$$
$$=-i+i=0$$

(vi) $\left(i^4+i^3+i^2+i\right)^2$ **10301016**

Solution:

$$\left[i^4+i^3+i^2+i\right]^2$$
$$=\left[\left(i^2\right)^2+i^2\times i+i^2+i\right]^2$$
$$=\left[(-1)^2+(-1)\times i+(-1)+i\right]^2 \qquad \because i^2=-1$$
$$=\left[1-i-1+i\right]^2$$
$$=[0]^2$$
$$=0$$

(vii) $\left(\frac{i^8}{i^5}\right)^{-5}$ **10301017**

Solution:

$$\left[\frac{i^8}{i^5}\right]^{-5}$$
$$=\left[\frac{i^5}{i^8}\right]^5 \qquad \because \left(\frac{a}{b}\right)^{-m}=\left(\frac{b}{a}\right)^m$$
$$=\frac{i^{25}}{i^{40}}=\frac{i^{24}\times i}{i^{40}}$$

$$=\frac{(i^2)^{12}\times i}{(i^2)^{20}}=\frac{(-1)^{12}\times i}{(-1)^{20}} \qquad \boxed{\because i^2=-1}$$

$$=\frac{1\times i}{1}=i$$

(viii) $i^{13}\times i^{29}$ **10301018**

Solution:

$$i^{13}\times i^{29}$$

$$=i^{13+29} \qquad \boxed{\because x^m\times x^n=x^{m+n}}$$

$$=i^{42}$$

$$=(i^2)^{21}=(-1)^{21} \qquad \boxed{\because i^2=-1}$$

$$=-1$$

2. Write in terms of i.

(i) $2+\sqrt{-4}$ **10301019**

Solution:

$$2+\sqrt{-4}$$

$$=2+\sqrt{4\times(-1)}$$

$$=2+\sqrt{4}\times\sqrt{-1}$$

$$=2+\sqrt{2^2}\times i \qquad \boxed{\because \sqrt{-1}=i}$$

$$=2+2i$$

(ii) $3-\sqrt{-7}$ **10301020**

Solution:

$$3-\sqrt{-7}$$

$$=3-\sqrt{7\times(-1)}$$

$$=3-\sqrt{7}\times\sqrt{-1}$$

$$=3-\sqrt{7}\times i \qquad \boxed{\because \sqrt{-1}=i}$$

$$=3-\sqrt{7}i$$

(iii) $\frac{2}{5}+\frac{\sqrt{-16}}{5}$ **10301021**

Solution:

$$\frac{2}{5}+\frac{\sqrt{-16}}{5}$$

$$=\frac{2}{5}+\frac{\sqrt{16\times(-1)}}{5}$$

$$=\frac{2}{5}+\frac{\sqrt{16}\times\sqrt{-1}}{5}$$

$$=\frac{2}{5}+\frac{\sqrt{4^2}\times i}{5} \qquad \boxed{\because \sqrt{-1}=i}$$

$$=\frac{2}{5}+\frac{4}{5}i$$

(iv) $\sqrt{2}-\sqrt{-3}$ **10301022**

Solution:

$$\sqrt{2}-\sqrt{-3}$$

$$=\sqrt{2}-\sqrt{3\times(-1)}$$

$$=\sqrt{2}-\sqrt{3}\times\sqrt{-1}$$

$$=\sqrt{2}-\sqrt{3}i \qquad \boxed{\because \sqrt{-1}=i}$$

3. Find the values of x and y.

(i) $(2x+5)+(y-3)i=1+2i$ **10301023**

Solution:

$$(2x+5)+(y-3)i=1+2i$$

By comparing real and imaginary parts of both sides, we get

$$2x+5=1 \quad \text{.........(i)}$$

$$y-3=2 \quad \text{.........(ii)}$$

From equation (i), we have

$$2x+5=1$$

$$2x=1-5$$

$$2x=-4$$

$$x=\frac{-4}{2}$$

$$x=-2$$

Now, from equation (ii), we have

$$y-3=2$$

$$y=2+3$$

$$y=5$$

So, $x=-2,\ y=5$

(ii) $(3x+2)-(4-y)i=5+3i$ **10301024**

Solution:

$$(3x+2)-(4-y)i=5+3i$$

By comparing real and imaginary parts of both sides, we have

$$3x+2=5 \quad \text{.........(i)}$$

$$-(4-y)=3 \quad \text{.........(ii)}$$

From equation (i), we have

$3x+2=5$

$3x=5-2$

$3x=3$

$x=\frac{3}{3}$

$x=1$

From equation (ii), we have:

$-(4-y)=3$

$-4+y=3$

$y=3+4$

$y=7$

So, $x=1,\ y=7$

(iii) $(2+i)x+(1-2i)y=3+4i$ **10301025**

Solution:

$(2+i)x+(1-2i)y=3+4i$

$2x+xi+y-2yi=3+4i$

$2x+y+xi-2yi=3+4i$

$(2x+y)+(x-2y)i=3+4i$

By comparing real and imaginary parts of both sides, we get

$2x+y=3$ ....(i)

$x-2y=4$ ....(ii)

Multiply equation (i) by '2' and Adding in equation (ii), we get

$$\begin{array}{r} 4x+\cancel{2y}=6 \\ x-\cancel{2y}=4 \\ \hline 5x=10 \end{array}$$

$x=\frac{10}{5}$

$x=2$

Put $x=2$ in equation (i), we have

$2(2)+y=3$

$4+y=3$

$y=3-4$

$y=-1$

So, $x=2,\ y=-1$

(iv) $(1-i)x+(2+i)y=4-i$ **10301026**

Solution:

$(1-i)x+(2+i)y=4-i$

$x-xi+2y+yi=4-i$

$x+2y-xi+yi=4-i$

$(x+2y)+(-x+y)i=4-i$

By comparing real and imaginary parts of both sides, we get

$x+2y=4$ ....(i)

$-x+y=-1$ ....(ii)

Adding equation (i) and (ii), we have

$$\begin{array}{r} \cancel{x}+2y=4 \\ \cancel{-x}+y=-1 \\ \hline 3y=3 \end{array}$$

$y=\frac{3}{3}$

$y=1$

Put $y=1$ in equation (i), we have

$x+2(1)=4$

$x+2=4$

$x=4-2$

$x=2$

So, $x=2,\ y=1$

(v) $(3x-1)+(2y-3)i=8+7i$ **10301027**

Solution:

$(3x-1)+(2y-3)i=8+7i$

By comparing real and imaginary parts of both sides, we get

$3x-1=8$ ....(i)

$2y-3=7$ ....(ii)

From equation (i), we have

$3x-1=8$

$3x=8+1$

$3x=9$

$x=\frac{9}{3}$

$x=3$

From equation (ii), we have

$$2y-3=7$$
$$2y=7+3$$
$$2y=10$$
$$y=\frac{10}{2}$$
$$y=5$$

So, $x=3,\ \ y=5$

Algebraic Operations on ComplexNumbers

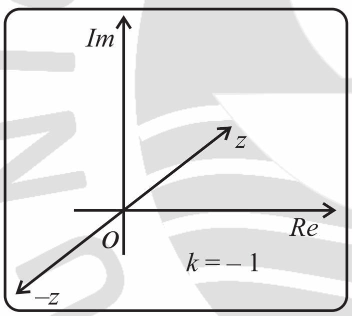
Q. What is the scalar multiplication of complex numbers? **10301028**

Ans. Scalar multiplication of Complex Numbers

If $z=x+iy$ and $k\in R$, then we define $kz=(kx)+(ky)i$ or $kx+i(ky)$

The following diagram shows kz for $k=-1,2,\frac{1}{2}$

(i)



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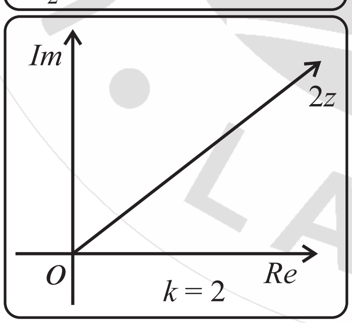
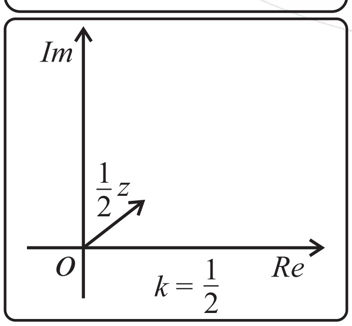


(iii)



Q. Describe the addition of two complex numbers. **10301029**

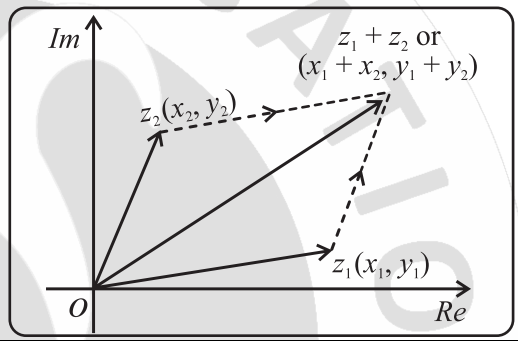
Ans. Addition of Two Complex Numbers

If $z_1=x_1+iy_1$ and $z_2=x_2+iy_2$ where x_1, x_2, y_1 and $y_2\in R$, then

$$z_1+z_2=(x_1+iy_1)+(x_2+iy_2)$$
$$=(x_1+x_2)+i(y_1+y_2)$$

When $z_1=x_1+iy_1$, and $z_2=x_2+iy_2$, then by the parallelogram law of addition,

$$z_1+z_2=(x_1+x_2)+i(y_1+y_2)$$



Q. Who formulated the algebra for complex numbers? **10301030**

Ans. History

The fundamental rules for addition, subtraction, multiplication and division of complex numbers were formulated by the Italian mathematician Rafael Bombelli (1526 – 1572). He is widely recognized as the first to establish a systematic algebra for complex numbers.

Example 3: Add $(3+4i)$ and $(5-2i)$

Solution: **10301031**

$$(3+4i)+(5-2i)$$
$$=(3+5)+(4-2)i$$
$$=8+2i$$

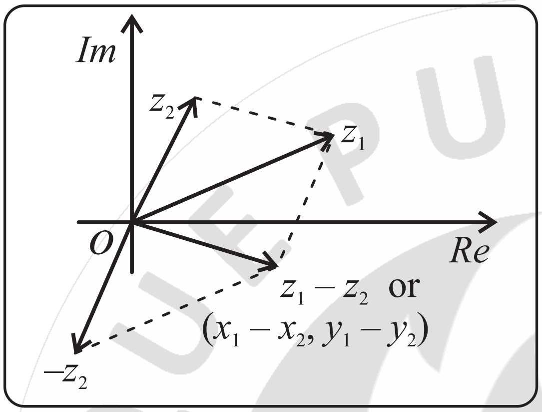
Q. Describe the subtraction of two complex numbers? **10301032**

Ans. Subtraction of Two Complex Numbers

If $z_1=x_1+iy_1$ and $z_2=x_2+iy_2$, where $x_1, x_2, y_1, y_2\in R$, then

$z_1 - z_2 = (x_1 + iy_1) - (x_2 + iy_2)$
$z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$
$z_1 - z_2 = z_1 + (-z_2)$

Following figure shows the subtraction of two complex numbers.



Example 4: Simplify:

(i) $(8+2i)-(5-6i)$ **10301033**

Solution:

$(8+2i)-(5-6i)$
$= 8+2i-5+6i$
$= (8-5)+(2i+6i)$
$= 3+8i$

(ii) $(4-3i)-(2-5i)$ **10301034**

Solution:

$(4-3i)-(2-5i)$
$= 4-3i-2+5i$
$= (4-2)+(-3+5)i$
$= 2+2i$

Skilled Practice

Q. If $z_1 - z_2 = 4+6i$ **and** $z_2 = 3-2i$ **Find** z_1.

Solution: **10301035**

Given: $z_1 - z_2 = 4+6i,\ z_2 = 3-2i$

$z_1 - (3-2i) = 4+6i$
$z_1 - 3 + 2i = 4+6i$
$z_1 = 4+3+6i-2i$
$z_1 = 7+4i$

Q. Describe the multiplication of two complex numbers. **10301036**

Ans. **Multiplication of Two Complex Numbers**

Let $z_1 = x_1 + iy_1, z_2 = x_2 + iy_2$, then

$z_1 z_2 = (x_1 + iy_1)(x_2 + iy_2)$
$= x_1(x_2 + iy_2) + iy_1(x_2 + iy_2)$
$= x_1x_2 + x_1y_2i + y_1x_2i + i^2 y_1y_2$ $\quad \because i^2 = -1$
$= (x_1x_2 - y_1y_2) + i(x_1y_2 + y_1x_2)$

Example 5:

(i) Simplify: $(3-4i)(5-6i)$ **10301037**

(ii) If $z_1 = 2+3i$ **and** $z_2 = 4+7$**, then find** z_1z_2. **10301038**

Solution:

(i) $(3-4i)(5-6i)$
$= 3(5-6i) - 4i(5-6i)$
$= 15 - 18i - 20i + 24i^2$
$= 15 - (18+20)i + 24(-1)$ $\quad \because i^2 = -1$
$= 15 - 38i - 24$
$= (15-24) - 38i$
$= -9 - 38i$

(ii) $z_1z_2 = (2+3i)(4+7i)$

Using

$(x_1 + iy_1)(x_2 + iy_2) = (x_1x_2 - y_1y_2)$
$\qquad + i(x_1y_2 + y_1x_2)$

$z_1z_2 = (2\times4 - 3\times7) + i(2\times7 + 3\times4)$
$= (8-21) + (14+12)i$
$= -13 + 26i$

Q. Describe the division of two complex numbers. **10301039**

Ans. **Division of Two Complex Numbers**

Let $z_1 = x_1 + iy_1, z_2 = x_2 + iy_2$

Now, $\dfrac{z_1}{z_2} = \dfrac{x_1 + iy_1}{x_2 + iy_2}, z_2 \neq 0$

$$=\frac{x_1+iy_1}{x_2+iy_2}\times\frac{x_2-iy_2}{x_2-iy_2}$$

$$=\frac{x_1x_2-ix_1y_2+iy_1x_2-i^2y_1y_2}{(x_2)^2-(iy_2)^2}$$

$$=\frac{x_1x_2+iy_1x_2-ix_1y_2+y_1y_2}{x_2^{\ 2}+y_2^2} \quad \boxed{\because i^2=-1}$$

$$=\frac{(x_1x_2+y_1y_2)+i(y_1x_2-x_1y_2)}{x_2^2+y_2^2}$$

$$=\frac{(x_1x_2+y_1y_2)}{x_2^2+y_2^2}+\frac{i(y_1x_2-x_1y_2)}{x_2^2+y_2^2}$$

Skilled Practice

1. Simplify: **10301040**

$$\left(\frac{1+2i}{3-i}\right)(2+i)$$

Solution:

$$\left(\frac{1+2i}{3-i}\right)(2+i)$$

$$=\frac{(1+2i)(2+i)}{3-i}$$

$$=\frac{1(2+i)+2i(2+i)}{3-i}$$

$$=\frac{2+i+4i+2i^2}{3-i}$$

$$=\frac{2+5i+2(-1)}{3-i} \quad \boxed{\because i^2=-1}$$

$$=\frac{2+5i-2}{3-i}$$

$$=\frac{5i}{3-i}\times\frac{3+i}{3+i}$$

$$=\frac{5i(3+i)}{(3-i)(3+i)}$$

$$=\frac{15i+5i^2}{(3)^2-i^2}$$

$$\boxed{\because (a+b)(a-b)=a^2-b^2}$$

$$=\frac{15i+5(-1)}{9-(-1)} \quad \boxed{\because i^2=-1}$$

$$=\frac{15i-5}{9+1}$$

$$=\frac{15i-5}{10}$$

$$=\frac{15}{10}i-\frac{5}{10}$$

$$=\frac{3}{2}i-\frac{1}{2}$$

$$=-\frac{1}{2}+\frac{3}{2}i$$

2. Find the complex number z: if $\frac{z}{2+i}=3-i$ **10301041**

Solution:

$$\frac{z}{2+i}=3-i$$

$$z=(3-i)(2+i)$$

$$z=3(2+i)-i(2+i)$$

$$z=6+3i-2i-i^2$$

$$z=6+i-(-1) \quad \boxed{\because i^2=-1}$$

$$z=6+i+1$$

$$z=7+i$$

Example 6: **Express** $\frac{3+4i}{5-7i}$ **in the form of** $x+iy$ **or** $x+yi$. **10301042**

Solution:

$$\frac{3+4i}{5-7i}$$

$$=\frac{3+4i}{5-7i}\times\frac{5+7i}{5+7i}$$

$$=\frac{15+21i+20i+28i^2}{(5)^2-(7i)^2}$$

$$=\frac{15+41i+28(-1)}{25-49i^2} \quad \boxed{\because i^2=-1}$$

$$=\frac{15+41i-28}{25-49(-1)}$$

$= \frac{-13+41i}{25+49}$

$= \frac{-13+41i}{74}$

$= \frac{-13}{74} + \frac{41}{74}i$

Properties of Complex Numbers

Q. State the closure properties of complex numbers. **10301043**

Ans. **Closure Property**

w.r.t Addition:

For any two complex numbers z_1 and z_2, the sum $z_1 + z_2$ is also a complex number.

w.r.t Multiplication:

For any two complex numbers z_1 and z_2, the product z_1z_2 is also a complex number.

Q. State the commutative properties of complex numbers? **10301044**

Ans. **Commutative Property**

w.r.t Addition:

For any two complex numbers z_1 and z_2,

$z_1 + z_2 = z_2 + z_1$

w.r.t Multiplication:

For any two complex numbers z_1 and z_2,

$z_1z_2 = z_2z_1$

Q. State associative properties of complex numbers. **10301045**

Ans. **Associative Property**

w.r.t Addition:

For any three complex numbers z_1, z_2 and z_3,

$(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$

w.r.t Multiplication:

For any three complex numbers z_1, z_2 and z_3,

$(z_1z_2)z_3 = z_1(z_2z_3)$

Q. State the additive identity of complex number. **10301046**

Ans. **Additive Identity of Complex Number**

There exists a complex number $0 = 0 + 0i$ Such that, for every complex number z,

$z + 0 = 0 + z = z$

The complex number $0 = 0 + 0i$ is known as additive identity.

Q. State the multiplicative identity of complex number. **10301047**

Ans. **Multiplicative Identity of Complex Number**

There exists a complex number $1 = 1 + 0i$ such that, for every complex number z,

$$z \times 1 = 1 \times z = z$$

The complex number $1 = 1 + 0i$ is known as multiplicative identity.

Q. State the additive inverse of a complex number. **10301048**

Ans. **The Additive Inverse**

For every complex number z there exists a complex number $-z$ such that,

$$z + (-z) = (-z) + z = 0.$$

$-z$ is called the additive inverse of z.

Q. State the multiplicative inverse of a complex number. **10301049**

Ans. **The Multiplicative Inverse**

For any non-zero complex number z, there exists a complex number w such that, $zw = wz = 1$

w is called the multiplicative inverse of z, and it is denoted by z^{-1}, here $w = z^{-1}$.

Q. State the distributive property of complex numbers. **10301050**

Ans. The distributive property of multiplication over addition is given below.

For any three complex numbers z_1, z_2 and z_3,

$$z_1(z_2 + z_3) = z_1z_2 + z_1z_3$$

and $(z_1 + z_2)z_3 = z_1z_3 + z_2z_3$

Example 7: **If $z = -7 - 4i$, then show that $z + 0 = z$.** **10301051**

Solution:

$z + 0 = (-7 - 4i) + (0 + 0i)$

$= -7 - 4i + 0 + 0i$

$= -7 - 4i = z$

It shows $z + 0 = z$ (0 is the additive identity)

Example 8: **Verify the multiplicative identity for** $z=3-2i$**.** 10301052

Solution:

$z\times 1=(3-2i)(1+0i)$

$=3+0i-2i-0i^2$

$=3-2i=z$

$1\times z=(1+0i)(3-2i)$

$=3-2i+0i-0i^2$

$=3-2i=z$

Hence, verified that $z\times 1=1\times z=z$

Example 9: **Find the additive inverse of** $z=7-10i$**.** 10301053

Solution: $z=7-10i$

Additive inverse of $z=-7+10i$

Because $z+(-z)=(7-10i)+(-7+10i)$

$=7-10i-7+10i=0$

Example 10: **Find multiplicative inverse of** $4-3i$**.** 10301054

Solution: Let $z=4-3i$ (given)

Then, $z^{-1}=\dfrac{1}{z}$

$$z^{-1}=\frac{1}{4-3i}$$

$$=\frac{4+3i}{(4-3i)(4+3i)}$$

$$=\frac{4+3i}{(4)^2-(3i)^2}$$

$$=\frac{4+3i}{16-9i^2}$$

$$=\frac{4+3i}{16-9(-1)} \qquad \because i^2=-1$$

$$z^{-1}=\frac{4+3i}{16+9}$$

$$z^{-1}=\frac{4+3i}{25}$$

$$z^{-1}=\frac{4}{25}+\frac{3i}{25}$$

EXERCISE 1.2

1. Simplify and write in the form $a+bi$:

(i) $(2+5i)+(3-zi)$ 10301055

Solution:

$(2+5i)+(3-zi)$

$=2+5i+3-zi$

$=2+3+5i-zi$

$=5+(5-z)i$

(ii) $(16-3i)+(9+2i)$ 10301056

Solution:

$(16-3i)+(9+2i)$

$=16-3i+9+2i$

$=16+9-3i+2i$

$=25-i$

(iii) $(9-2i)-(7-3i)$ 10301057

Solution:

$(9-2i)-(7-3i)$

$=9-2i-7+3i$

$=9-7-2i+3i$

$=2+i$

(iv) $(11+9i)-(9-7i)$ 10301058

Solution:

$(11+9i)-(9-7i)$

$=11+9i-9+7i$

$=11-9+9i+7i$

$=2+16i$

(v) $(3+4i)(2-3i)$ 10301059

Solution:

$(3+4i)(2-3i)$

$=3(2-3i)+4i(2-3i)$

$=6-9i+8i-12i^2$

$=6-i-12(-1)$ $\quad \because i^2=-1$

$=6-i+12$

$=18-i$

(vi) $(5-2i)(3-4i)$ **10301060**

Solution:

$(5-2i)(3-4i)$

$=5(3-4i)-2i(3-4i)$

$=15-20i-6i+8i^2$

$=15-26i+8(-1)$ $\quad \because i^2=-1$

$=15-26i-8$

$=7-26i$

(vii) $(3-5i)\div(2-4i)$ **10301061**

Solution:

$(3-5i)\div(2-4i)$

$=\dfrac{3-5i}{2-4i}$

$=\dfrac{(3-5i)}{(2-4i)}\times\dfrac{(2+4i)}{(2+4i)}$

$=\dfrac{3(2+4i)-5i(2+4i)}{(2)^2-(4i)^2}$

$\because (a+b)(a-b)=a^2-b^2$

$=\dfrac{6+12i-10i-20i^2}{4-16i^2}$

$=\dfrac{6+2i-20(-1)}{4-16(-1)}$ $\quad \because i^2=-1$

$=\dfrac{6+2i+20}{4+16}$

$=\dfrac{26+2i}{20}$

$=\dfrac{26}{20}+\dfrac{2}{20}i$

$=\dfrac{13}{10}+\dfrac{1}{10}i$

(viii) $(5+2i)\div(6-3i)$ **10301062**

Solution:

$(5+2i)\div(6-3i)$

$=\dfrac{(5+2i)}{(6-3i)}$

$=\dfrac{(5+2i)}{(6-3i)}\times\dfrac{(6+3i)}{(6+3i)}$

$=\dfrac{5(6+3i)+2i(6+3i)}{(6)^2-(3i)^2}$

$\because (a+b)(a-b)=a^2-b^2$

$=\dfrac{30+15i+12i+6i^2}{36-9i^2}$

$=\dfrac{30+27i+6(-1)}{36-9(-1)}$ $\quad \because i^2=-1$

$=\dfrac{30+27i-6}{36+9}$

$=\dfrac{24+27i}{45}$

$=\dfrac{24}{45}+\dfrac{27}{45}i$

$=\dfrac{8}{15}+\dfrac{3}{5}i$

2. Write additive inverse for each complex number:

(i) $3+2i$ **10301063**

Solution:

Let $z=3+2i$

Additive Inverse of $z=-z=-(3+2i)$

$-z=-3-2i$

(ii) $4-3i$ **10301064**

Solution:

Let $z=4-3i$

Additive Inverse of $z=-z=-(4-3i)$

$-z=-4+3i$

(iii) $5-7i$ **10301065**

Solution:

Let $z=5-7i$

Additive Inverse of $z=-z=-(5-7i)$

$-z=-5+7i$

(iv) $\frac{-2}{3}+\frac{5}{4}i$ **10301066**

Solution:

Let $z=\frac{-2}{3}+\frac{5}{4}i$

Additive Inverse of $z=-z=-\left[\frac{-2}{3}+\frac{5}{4}i\right]$

$-z=\frac{2}{3}-\frac{5}{4}i$

3. Find multiplicative inverse for each complex number:

(i) $4+5i$ **10301067**

Solution:

$4+5i$

Let $z=4+5i$

Then, $z^{-1}=\frac{1}{z}$

$$z^{-1}=\frac{1}{4+5i}$$
$$=\frac{1}{4+5i}\times\frac{4-5i}{4-5i}$$
$$=\frac{4-5i}{(4+5i)(4-5i)}$$
$$=\frac{4-5i}{(4)^2-(5i)^2}$$

$\boxed{\because (a+b)(a-b)=a^2-b^2}$

$$=\frac{4-5i}{16-25i^2}$$
$$=\frac{4-5i}{16-25(-1)} \quad \boxed{\because i^2=-1}$$
$$=\frac{4-5i}{16+25}=\frac{4-5i}{41}$$
$$z^{-1}=\frac{4}{41}-\frac{5}{41}i$$

(ii) $6+2i$ **10301068**

Solution:

Let $z=6+2i$

Then, $z^{-1}=\frac{1}{z}$

$$z^{-1}=\frac{1}{6+2i}$$
$$=\frac{1}{6+2i}\times\frac{6-2i}{6-2i}$$
$$=\frac{6-2i}{(6+2i)(6-2i)}$$
$$=\frac{6-2i}{(6)^2-(2i)^2}$$

$\boxed{\because (a+b)(a-b)=a^2-b^2}$

$$=\frac{6-2i}{36-4i^2}$$
$$=\frac{6-2i}{36-4(-1)} \quad \boxed{\because i^2=-1}$$
$$=\frac{6-2i}{36+4}$$
$$=\frac{6-2i}{40}$$
$$=\frac{6}{40}-\frac{2}{40}i$$
$$z^{-1}=\frac{3}{20}-\frac{1}{20}i$$

(iii) $7-3i$ **10301069**

Solution:

Let $z=7-3i$

Then, $z^{-1}=\frac{1}{z}$

$$z^{-1}=\frac{1}{7-3i}$$
$$=\frac{1}{7-3i}\times\frac{7+3i}{7+3i}$$
$$=\frac{7+3i}{(7-3i)(7+3i)}$$
$$=\frac{7+3i}{(7)^2-(3i)^2}$$

$\boxed{\because (a+b)(a-b)=a^2-b^2}$

$$=\frac{7+3i}{49-9i^2}$$

$$=\frac{7+3i}{49-9(-1)} \qquad \boxed{\because i^2=-1}$$

$$=\frac{7+3i}{49+9}$$

$$=\frac{7+3i}{58}$$

$$z^{-1}=\frac{7}{58}+\frac{3}{58}i$$

(iv) $\sqrt{5}-4i$ 10301070

Solution:

Let $z=\sqrt{5}-4i$

Then, $z^{-1}=\frac{1}{z}$

$$z^{-1}=\frac{1}{\sqrt{5}-4i}$$

$$=\frac{1}{\sqrt{5}-4i}\times\frac{\sqrt{5}+4i}{\sqrt{5}+4i}$$

$$=\frac{\sqrt{5}+4i}{\left(\sqrt{5}-4i\right)\left(\sqrt{5}+4i\right)}$$

$$=\frac{\sqrt{5}+4i}{\left(\sqrt{5}\right)^2-(4i)^2}$$

$$\boxed{\because (a+b)(a-b)=a^2-b^2}$$

$$=\frac{\sqrt{5}+4i}{5-16i^2}$$

$$=\frac{\sqrt{5}+4i}{5-16(-1)} \qquad \boxed{\because i^2=-1}$$

$$=\frac{\sqrt{5}+4i}{5+16}$$

$$=\frac{\sqrt{5}+4i}{21}$$

$$z^{-1}=\frac{\sqrt{5}}{21}+\frac{4}{21}i$$

4. If $z_1=2+5i, z_2=1-3i$ **and** $z_3=2+i$, **then verify that**

(i) $z_1+z_2=z_2+z_1$ 10301071

Solution:

Given that: $z_1=2+5i, z_2=1-3i$

L.H.S. $= z_1+z_2$

$=(2+5i)+(1-3i)$

$=2+5i+1-3i$

$=3+2i$ ....(i)

R.H.S. $=z_2+z_1$

$=(1-3i)+(2+5i)$

$=1-3i+2+5i$

$=3+2i$ ....(ii)

From (i) and (ii)

L.H.S. = R.H.S.

$z_1+z_2=z_2+z_1$. Hence proved.

(ii) $z_1z_2=z_2z_1$ 10301072

Solution:

Given that: $z_1=2+5i, z_2=1-3i$

L.H.S. $=z_1z_2$

$=(2+5i)(1-3i)$

$=2(1-3i)+5i(1-3i)$

$=2-6i+5i-15i^2$

$=2-i-15(-1)$ $\boxed{\because i^2=-1}$

$=2-i+15$

$=17-i$ ....(i)

R.H.S. $=z_2z_1$

$=(1-3i)(2+5i)$

$=1(2+5i)-3i(2+5i)$

$=2+5i-6i-15i^2$

$=2-i-15(-1)$ $\boxed{\because i^2=-1}$

$=2-i+15$

$=17-i$ ....(ii)

From equation (i) and (ii), we have

L.H.S. = R.H.S.

$z_1z_2=z_2z_1$. Hence proved.

(iii) $(z_1+z_2)+z_3=z_1+(z_2+z_3)$ **10301073**

Solution:

Given that: $z_1=2+5i, z_2=1-3i, z_3=2+i$

L.H.S. $=(z_1+z_2)+z_3$

$=[(2+5i)+(1-3i)]+(2+i)$

$=(2+5i+1-3i)+(2+i)$

$=3+2i+2+i$

$=5+3i$ ....(i)

R.H.S. $=z_1+(z_2+z_3)$

$=(2+5i)+[(1-3i)+(2+i)]$

$=(2+5i)+(1-3i+2+i)$

$=2+5i+3-2i$

$=5+3i$ ....(ii)

From equation (i) and (ii), we have

L.H.S. = R.H.S.

$(z_1+z_2)+z_3=z_1+(z_2+z_3)$. Hence proved.

(iv) $(z_1z_2)z_3=z_1(z_2z_3)$ **10301074**

Solution:

Given that: $z_1=2+5i, z_2=1-3i, z_3=2+i$

L.H.S. $=(z_1z_2)z_3$

$=[(2+5i)(1-3i)](2+i)$

$=[2(1-3i)+5i(1-3i)](2+i)$

$=(2-6i+5i-15i^2)(2+i)$

$=[2-i-15(-1)](2+i)$ $\boxed{\because i^2=-1}$

$=(2-i+15)(2+i)$

$=(17-i)(2+i)$

$=17(2+i)-i(2+i)$

$=34+17i-2i-i^2$

$=34+15i-(-1)$ $\boxed{\because i^2=-1}$

$=34+15i+1$

$=35+15i$ ....(i)

R.H.S. $=z_1(z_2z_3)$

$=(2+5i)[(1-3i)(2+i)]$

$=(2+5i)[1(2+i)-3i(2+i)]$

$=(2+5i)(2+i-6i-3i^2)$

$=(2+5i)(2-5i-3(-1))$ $\boxed{\because i^2=-1}$

$=(2+5i)(2-5i+3)$

$=(2+5i)(5-5i)$

$=2(5-5i)+5i(5-5i)$

$=10-10i+25i-25i^2$

$=10+15i-25(-1)$ $\boxed{\because i^2=-1}$

$=10+15i+25$

$=35+15i$ ....(ii)

From equation (i) and (ii), we have

L.H.S. = R.H.S.

$(z_1z_2)z_3=z_1(z_2z_3)$. Hence proved.

(v) $z_1+(-z_1)=(-z_1)+z_1=0$ **10301075**

Solution:

Given that:

$z_1=2+5i$

$-z_1=-(2+5i)=-2-5i$

Now,

$z_1+(-z_1)=(2+5i)+(-2-5i)$

$=2+5i-2-5i$

$=0$ ....(i)

Also,

$(-z_1)+z_1=(-2-5i)+(2+5i)$

$=-2-5i+2+5i$

$=0$ ....(ii)

From equation (i) and (ii), we have

$z_1+(-z_1)=(-z_1)+z_1=0$

Hence proved.

5. If $\frac{(1+i)^2}{2-i} = x+iy$, then find the values of x and y. 10301076

Solution:

$$\frac{(1+i)^2}{2-i} = x+iy$$

$$\frac{1^2+i^2+2(1)(i)}{2-i} = x+iy$$

$$\because (a+b)^2 = a^2+b^2+2ab$$

$$\frac{1+(-1)+2i}{2-i} = x+iy$$

$$\frac{1-1+2i}{2-i} = x+iy$$

$$\frac{2i}{2-i} = x+iy$$

$$\frac{2i}{2-i}\times\frac{2+i}{2+i} = x+iy$$

$$\frac{4i+2i^2}{(2)^2-(i)^2} = x+iy$$

$$\because (a+b)(a-b) = a^2-b^2$$

$$\frac{4i+2i^2}{4-i^2} = x+iy$$

$$\frac{4i+2(-1)}{4-(-1)} = x+iy \qquad \because i^2=-1$$

$$\frac{4i-2}{4+1} = x+iy$$

$$\frac{-2+4i}{5} = x+iy$$

$$\frac{-2}{5}+\frac{4}{5}i = x+iy$$

By comparing real and imaginary parts of both sides, we have

$$\Rightarrow x = \frac{-2}{5}, \qquad y = \frac{4}{5}$$

6. If $(2x+yi)(1-i) = 4+2i$, then find the values of x and y. 10301077

Solution:

$$(2x+iy)(1-i) = 4+2i$$

$$2x+iy = \frac{4+2i}{1-i}$$

$$2x+iy = \frac{(4+2i)}{(1-i)}\times\frac{(1+i)}{(1+i)}$$

$$2x+iy = \frac{4(1+i)+2i(1+i)}{(1)^2-(i)^2}$$

$$\because (a+b)(a-b) = a^2-b^2$$

$$2x+iy = \frac{4+4i+2i+2i^2}{1-i^2}$$

$$2x+iy = \frac{4+6i+2(-1)}{1-(-1)} \qquad \because i^2=-1$$

$$2x+iy = \frac{4+6i-2}{1+1}$$

$$2x+iy = \frac{2+6i}{2}$$

$$2x+iy = \frac{2}{2}+\frac{6}{2}i$$

$$2x+iy = 1+3i$$

By comparing real and imaginary parts of both sides, we have

$$2x = 1, \quad y = 3$$

$$x = \frac{1}{2}$$

7. Find the values of a and b if $(a+bi)(1+3i) = -8+11i$. 10301078

Solution:

$$a+bi = \frac{-8+11i}{1+3i}$$

$$a+bi = \frac{(-8+11i)}{(1+3i)}\times\frac{(1-3i)}{(1-3i)}$$

$$a+bi=\frac{-8(1-3i)+11i(1-3i)}{(1)^2-(3i)^2}$$

$\because a^2-b^2=(a+b)(a-b)$

$$a+bi=\frac{-8+24i+11i-33i^2}{1-9i^2}$$

$$a+bi=\frac{-8+35i-33(-1)}{1-9(-1)} \qquad \because i^2=-1$$

$$a+bi=\frac{-8+35i+33}{1+9}$$

$$a+bi=\frac{25+35i}{10}$$

$$a+bi=\frac{25}{10}+\frac{35}{10}i$$

$$a+bi=\frac{5}{2}+\frac{7}{2}i$$

By comparing real and imaginary parts of both sides, we have

$$\Rightarrow a=\frac{5}{2},\ b=\frac{7}{2}$$

Q. Define complex or argand plane.

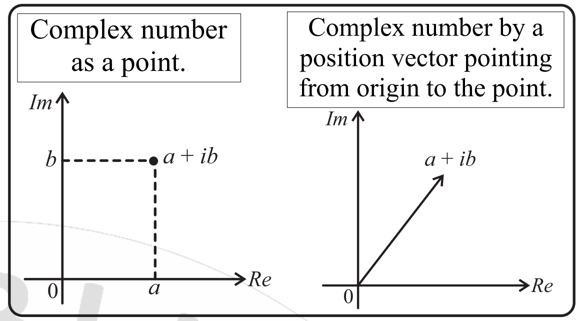
Ans. <u>Complex or Argand Plane</u> **10301079**

A complex number $z=x+iy$ can be represented by the point (x,y) in the coordinate plane.

If we consider $x-axis$ as reals axis and $y-axis$ as imaginary axis to represent a complex number, then the $xy-plane$ is called complex plane or Argand plane. It is named in honour of Swiss mathematician Jean Argand (1768 – 1822).

Q. Describe the complex number as a point and as a vector. **10301080**

Ans. A complex number is represented not only by a point, but also by a position vector pointing from the origin to the point. The complex number, the corresponding point and the vector are all typically denoted by the same symbol, z. Geometrically, a complex number can be interpreted either as a point in the complex plane C or as a vector in the Argand plane.

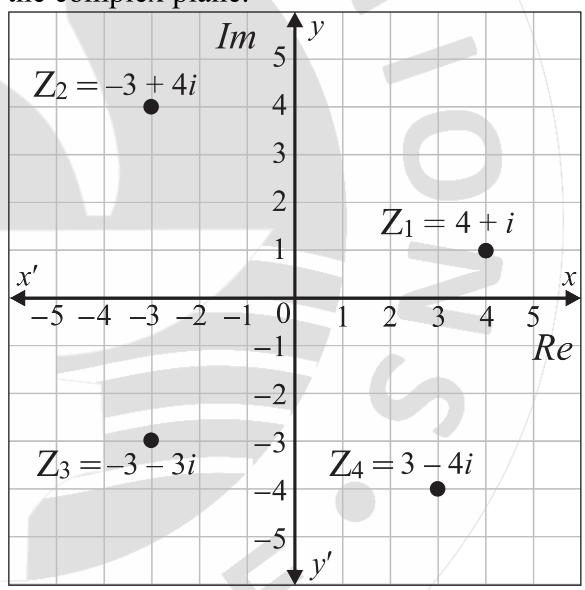


Q. Plot the following complex numbers on the complex plane. **10301081**

(i) $z_1=4+i$ **(ii)** $z_2=-3+4i$

(iii) $z_3=-3-3i$ **(iv)** $z_4=3-4i$

Here some complex numbers are plotted on the complex plane.



Q. What is the conjugate of a complex number? Give an example. **10301082**

Ans. <u>Conjugate of a Complex Number</u>

The conjugate of the complex number $x+iy$ is defined as the complex number $x-iy$. The complex conjugate of z denoted by $\overline{z}$. To get the conjugate of the complex number z, simply change i by $-i$ in z. For instance $3-8i$ is the conjugate of $3+8i$. The product of a complex number and its conjugate is a real number.

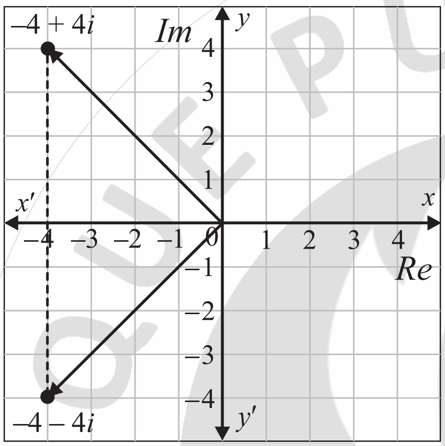
Q. Give the geometrical representation of the conjugates of the following complex numbers. **10301083**

(i) $z_1 = -4+4i$ **(ii)** $z_2 = x+iy = 3+3i$

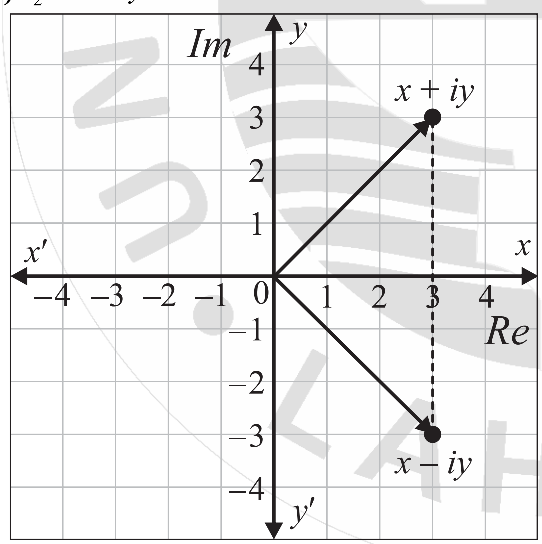
Ans. **<u>Geometrical Representation of Conjugate of a Complex Number</u>**

Geometrically, the conjugate of Z is obtained by reflecting Z on the real axis.

(i) $z_1 = -4+4i$



(ii) $z_2 = x+iy = 3+3i$



Q. How is the conjugate useful in the division? **10301084**

Ans. Conjugates are particularly useful when dividing complex numbers. By multiplying both the numerator and the denominator by the conjugate of the denominator, the complex number in the denominator can be transformed into a real number.

Properties of Complex Conjugate

For any two complex numbers z_1 and z_2, we have

Property 1: $\overline{z_1+z_2} = \overline{z_1}+\overline{z_2}$

Proof: Let $z_1 = x_1+iy_1, z_2 = x_2+iy_2$, where $x_1, y_1, x_2, y_2 \in R$

Now,

$$\overline{z_1+z_2} = \overline{(x_1+iy_1)+(x_2+iy_2)}$$
$$= \overline{(x_1+x_2)+i(y_1+y_2)}$$
$$= (x_1+x_2)-i(y_1+y_2)$$
$$= x_1+x_2-iy_1-iy_2$$
$$= (x_1-iy_1)+(x_2-iy_2) = \overline{z_1}+\overline{z_2}$$

Example 11: Let $z_1 = 4+3i$**,** $z_2 = 5+2i$**, then prove that** $\overline{z_1+z_2} = \overline{z_1}+\overline{z_2}$**.** **10301085**

Solution: $z_1+z_2 = 4+3i+5+2i = 9+5i$

$$\overline{z_1+z_2} = 9-5i \quad \text{.........(i)}$$
$$\overline{z_1} = 4-3i$$
$$\overline{z_2} = 5-2i$$
$$\overline{z_1}+\overline{z_2} = 4-3i+5-2i$$
$$\overline{z_1}+\overline{z_2} = 9-5i \quad \text{.........(ii)}$$

From (i) and (ii), we get

$$\overline{z_1+z_2} = \overline{z_1}+\overline{z_2}$$

Property 2: $\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$

Proof: Let $z_1 = (x_1+iy_1)$ and $z_2 = (x_2+iy_2)$ where $x_1, y_1, x_2, y_2 \in R$

$$z_1 z_2 = (x_1+iy_1)(x_2+iy_2)$$
$$= x_1x_2 + ix_1y_2 + iy_1x_2 + i^2y_1y_2$$
$$= (x_1x_2 - y_1y_2) + i(x_1y_2 + y_1x_2)$$

Therefore:

$$\overline{z_1 z_2} = \overline{(x_1x_2 - y_1y_2) + i(x_1y_2 + y_1x_2)}$$
$$= (x_1x_2 - y_1y_2) - i(x_1y_2 + y_1x_2) \quad \text{.........(i)}$$

and

$$\overline{z_1}\cdot\overline{z_2} = \left(\overline{x_1+iy_1}\right)\left(\overline{x_2+iy_2}\right)$$
$$= (x_1-iy_1)(x_2-iy_2)$$
$$= x_1x_2 - ix_1y_2 - iy_1x_2 + i^2y_1y_2$$
$$= (x_1x_2 - y_1y_2) - i(x_1y_2 + y_1x_2) \quad \text{......(ii)}$$

From (i) and (ii), we get

$$\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$$

Example 12: **If** $z_1 = 3+4i, z_2 = 2+3i$, **then prove that** $\overline{z_1 z_2} = \overline{z_1}\cdot\overline{z_2}$. **10301086**

Solution:

$$z_1 z_2 = (3+4i)(2+3i)$$
$$= 6+9i+8i+12i^2$$
$$= 6+17i-12 \qquad \because i^2=-1$$
$$= -6+17i$$
$$\overline{z_1 z_2} = -6-17i \qquad \ldots\ldots\ldots(i)$$

and
$$\overline{z_1}\cdot\overline{z_2} = \left(\overline{3+4i}\right)\left(\overline{2+3i}\right)$$
$$= (3-4i)(2-3i)$$
$$= 6-9i-8i+12i^2$$
$$= 6-9i-8i-12$$
$$\overline{z_1}\cdot\overline{z_2} = -6-17i \qquad \ldots\ldots\ldots(ii)$$

From (i) and (ii), we get

$$\therefore \quad \overline{z_1 z_2} = \overline{z_1}\cdot\overline{z_2}$$

Property 3: $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$

Proof: Let $z_1 = x_1 + iy_1$

and $z_2 = x_2 + iy_2$

Now, $\frac{z_1}{z_2} = \frac{x_1+iy_1}{x_2+iy_2}$

$$\frac{z_1}{z_2} = \frac{x_1+iy_1}{x_2+iy_2}\times\frac{x_2-iy_2}{x_2-iy_2}$$
$$\frac{z_1}{z_2} = \frac{(x_1x_2+y_1y_2)-i(x_1y_2-y_1y_2)}{x_2^2+y_2^2}$$

and
$$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{(x_1x_2+y_1y_2)+i(x_1y_2-y_1x_2)}{x_2^2+y_2^2} \ldots(i)$$

Now,
$$\frac{\overline{z_1}}{\overline{z_2}} = \frac{x_1-iy_1}{x_2-iy_2}$$
$$= \frac{(x_1-iy_1)(x_2+iy_2)}{(x_2-iy_2)(x_2+iy_2)}$$
$$= \frac{x_1x_2+ix_1y_2-iy_1x_2-i^2y_1y_2}{x_2^2+y_2^2}$$
$$\frac{\overline{z_1}}{\overline{z_2}} = \frac{(x_1x_2+y_1y_2)+i(x_1y_2-y_1x_2)}{x_2^2+y_2^2} \ldots\ldots(ii)$$

$\therefore$ from (i) and (ii) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$

Example 13: **If** $z_1 = 5+4i$, $z_2 = 3+2i$, **then prove that** $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$. **10301087**

Solution:

$$\frac{z_1}{z_2} = \frac{5+4i}{3+2i}$$
$$= \frac{(5+4i)}{(3+2i)}\times\frac{(3-2i)}{(3-2i)}$$
$$= \frac{15-10i+12i-8i^2}{(3)^2-(2i)^2}$$
$$= \frac{15+2i-8(-1)}{9-4i^2}$$
$$= \frac{15+2i+8}{9-4(-1)}$$
$$= \frac{23+2i}{9+4}$$
$$\frac{z_1}{z_2} = \frac{23+2i}{13}$$
$$\frac{z_1}{z_2} = \frac{23}{13}+\frac{2}{13}i$$

and
$$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{23}{13}-\frac{2}{13}i \qquad \ldots\ldots\ldots(i)$$

Now,
$$\frac{\overline{z_1}}{\overline{z_2}} = \frac{5-4i}{3-2i} = \frac{(5-4i)(3+2i)}{(3-2i)(3+2i)}$$
$$= \frac{15+10i-12i-8i^2}{(3)^2-(2i)^2}$$
$$= \frac{15-2i-8(-1)}{9-4i^2}$$

$$= \frac{15-2i+8}{9-4(-1)}$$

$$= \frac{23-2i}{9+4}$$

$$\frac{\overline{z_1}}{\overline{z_2}} = \frac{23-2i}{13}$$

$$\frac{\overline{z_1}}{\overline{z_2}} = \frac{23}{13} - \frac{2}{13}i \quad \text{.........(ii)}$$

From (i) and (ii), we get

$$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$$

Skilled Practice

Q. Take any two complex numbers and prove that:

(i) $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$ **10301088**

Solution:

Let $z_1 = 1+2i,\ z_2 = 1+3i$

L.H.S. $= (1+2i)+(1+3i)$

$= 1+2i+1+3i$

$z_1 + z_2 = 2+5i$

Taking conjugate on both sides, we have

$$\overline{z_1 + z_2} = \overline{2+5i}$$

$$\overline{z_1 + z_2} = 2-5i \quad \text{.........(i)}$$

R.H.S. $= \overline{z_1} + \overline{z_2}$

$z_1 = 1+2i \Rightarrow \overline{z_1} = 1-2i$

$z_2 = 1+3i \Rightarrow \overline{z_2} = 1-3i$

Now, $\overline{z_1} + \overline{z_2} = (1-2i)+(1-3i)$

$= 1-2i+1-3i$

$\overline{z_1} + \overline{z_2} = 2-5i$ ....(ii)

From equation (i) and (ii), we have

L.H.S. = R.H.S.

$$\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$$

(ii) $\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$ **10301089**

Solution:

$z_1 z_2 = (1+2i)(1+3i)$

$= 1(1+3i)+2i(1+3i)$

$= 1+3i+2i+6i^2$

$= 1+5i+6(-1)$ $\quad \boxed{\because i^2 = -1}$

$= 1+5i-6$

$z_1 z_2 = -5+5i$

Taking conjugate on both sides, we have

$$\overline{z_1 z_2} = \overline{-5+5i}$$

$$\overline{z_1 z_2} = -5-5i \quad \text{.........(i)}$$

R.H.S. $= \overline{z_1}\,\overline{z_2}$

$z_1 = 1+2i \Rightarrow \overline{z_1} = 1-2i$

$z_2 = 1+3i \Rightarrow \overline{z_2} = 1-3i$

Now, $\overline{z_1} \cdot \overline{z_2} = (1-2i)(1-3i)$

$= 1(1-3i)-2i(1-3i)$

$= 1-3i-2i+6i^2$

$= 1-5i+6(-1)$ $\quad \boxed{\because i^2 = -1}$

$= 1-5i-6$

$\overline{z_1} \cdot \overline{z_2} = -5-5i$ ....(ii)

From equation (i) and (ii), we have

L.H.S. = R.H.S

$\overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$ Hence proved.

(iii) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}, \quad (z_2 \neq 0)$ **10301090**

Solution:

$$\frac{z_1}{z_2} = \frac{1+2i}{1+3i}$$

$$= \frac{1+2i}{1+3i} \times \frac{1-3i}{1-3i}$$

$$= \frac{1(1-3i)+2i(1-3i)}{(1)^2-(3i)^2}$$

$$\boxed{\because (a+b)(a-b) = a^2-b^2}$$

$$= \frac{1-3i+2i-6i^2}{1-9i^2}$$

$$= \frac{1-i-6(-1)}{1-9(-1)} \quad \boxed{\because i^2=-1}$$

$$= \frac{1-i+6}{1+9}$$

$$= \frac{7-i}{10}$$

$$\frac{z_1}{z_2} = \frac{7}{10} - \frac{1}{10}i$$

Taking conjugate on both sides, we have

$$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{7}{10} + \frac{1}{10}i \quad \ldots\ldots\ldots(i)$$

R.H.S. $= \frac{\overline{z}_1}{\overline{z}_2}$

$$z_1 = 1+2i \Rightarrow \overline{z}_1 = 1-2i$$

$$z_2 = 1+3i \Rightarrow \overline{z}_2 = 1-3i$$

Now, $\frac{\overline{z}_1}{\overline{z}_2} = \frac{1-2i}{1-3i} \times \frac{1+3i}{1+3i}$

$$= \frac{(1-2i)(1+3i)}{(1-3i)(1+3i)}$$

$$= \frac{1(1+3i)-2i(1+3i)}{(1)^2-(3i)^2}$$

$$\boxed{\because (a+b)(a-b) = a^2-b^2}$$

$$= \frac{1+3i-2i-6i^2}{1-9i^2}$$

$$= \frac{1+i-6(-1)}{1-9(-1)} \quad \boxed{\because i^2=-1}$$

$$= \frac{1+i+6}{1+9}$$

$$= \frac{7+i}{10}$$

$$\frac{\overline{z}_1}{\overline{z}_2} = \frac{7}{10} + \frac{1}{10}i \quad \ldots\ldots\ldots(ii)$$

From equation (i) and (ii), we have

L.H.S. = R.H.S.

$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z}_1}{\overline{z}_2}$. Hence proved.

Property 4: $\overline{\overline{z}} = z$

Proof: Let $z = x+iy$

Then $\overline{z} = x-iy$

$$\overline{\overline{z}} = x+iy$$

$$\overline{\overline{z}} = z$$

Example 14: If $z = 7+3i$, then prove that $\overline{\overline{z}} = z$. **10301091**

Solution:

$$z = 7+3i$$

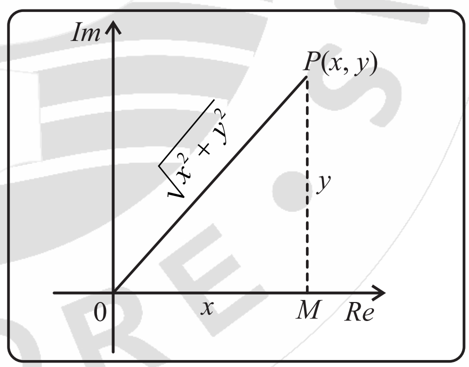
Then, $\overline{z} = 7-3i$

$$\overline{\overline{z}} = 7+3i = z$$

Hence, proved $\overline{\overline{z}} = z$

Modulus of a Complex Number

The modulus of a complex number measures the distance from the origin in the complex plane. Notice that the distance from the origin to the point $P(x,y)$ lies along a radial line and forms the hypotenuse of a right triangle, where the horizontal and vertical sides have length x and y respectively.



Q. Define the modulus of a complex number. Give at least one example. **10301092**

Ans. If $z = x+iy$, then the modulus of z is denoted by $|z|$ and defined as $|z| = \sqrt{x^2+y^2}$.

For example,

(i) $|i| = |0+1i| = \sqrt{0^2+1^2} = 1$

(ii) $|3-5i| = \sqrt{(3)^2+(-5)^2} = \sqrt{9+25} = \sqrt{34}$

Properties of Modulus of a Complex Number

Property 1: $|z| = |\bar{z}|$

Proof: Let $z = x + iy$

Then $|z| = \sqrt{x^2 + y^2}$ ....(i)

Now $\bar{z} = x - iy$

$|\bar{z}| = \sqrt{(x)^2 + (-y)^2} = \sqrt{x^2 + y^2}$ ....(ii)

From (i) and (ii), we get

Therefore, $|\bar{z}| = |z|$ From (i) and (ii)

Example 15: If $z = 5 + 4i$, **show that** $|z| = |\bar{z}|$

Solution: **10301093**

$z = 5 + 4i$

$|z| = \sqrt{(5)^2 + (4)^2}$

$|z| = \sqrt{25 + 16} = \sqrt{41}$ ....(i)

Now, $\bar{z} = 5 - 4i$

$|\bar{z}| = \sqrt{(5)^2 + (-4)^2}$

$|\bar{z}| = \sqrt{25 + 16} = \sqrt{41}$ ....(ii)

From (i) and (ii), we get

$|z| = |\bar{z}|$

Property 2: $|z| = |-z| = |\bar{\bar{z}}| = |-\bar{z}|$

Proof: Let $z = x + iy$

Then $|z| = \sqrt{x^2 + y^2}$ ....(i)

$|-z| = |-x - iy|$

$|-z| = \sqrt{(-x)^2 + (-y)^2}$

$|-z| = \sqrt{x^2 + y^2}$ ....(ii)

Now, $\bar{z} = x - yi$

$\Rightarrow \bar{\bar{z}} = x + yi$

and $|\bar{\bar{z}}| = \sqrt{x^2 + y^2}$ ....(iii)

Here $\bar{z} = x - iy$ as $z = x + iy$

$-\bar{z} = -x + iy$

$|-\bar{z}| = \sqrt{(-x)^2 + (y)^2}$

$|-\bar{z}| = \sqrt{x^2 + y^2}$ ....(iv)

From (i), (ii), (iii) and (iv), we have

$|z| = |-z| = |\bar{\bar{z}}| = |-\bar{z}|$

Property 3: $z\bar{z} = |z|^2$

Proof: Let $z = x + iy$

$\therefore$ $\bar{z} = x - iy$

Now, $z\bar{z} = (x + iy)(x - iy)$

$= (x)^2 - (iy)^2$

$\boxed{\because (a+b)(a-b) = a^2 - b^2}$

$= x^2 - (i^2 y^2)$

$= x^2 - (-y^2)$ as $i^2 = -1$

$= x^2 + y^2 = |z|^2$ as $|z| = \sqrt{x^2 + y^2}$

Hence, $z\bar{z} = |z|^2$

Example 16: If $z = 5 + 3i$, show that $z\bar{z} = |z|^2$

Solution: **10301094**

$z = 5 + 3i$

$\bar{z} = 5 - 3i$

$z\bar{z} = (5 + 3i)(5 - 3i)$

$= (5)^2 - (3i)^2$

$= 25 - (9i^2)$

$= 25 - (-9)$ $\boxed{\because i^2 = -1}$

$= 25 + 9 = 34$ ....(i)

As $z = 5 + 3i$

$|z| = \sqrt{(5)^2 + (3)^2}$

$|z|^2 = \left[\sqrt{(5)^2 + (3)^2}\right]^2$

$= 25 + 9 = 34$ ....(ii)

From (i) and (ii), we get

$z\bar{z} = |z|^2$

EXERCISE 1.3

1. Find the modulus of the following complex numbers:

(i) $4+3i$ 10301095

Solution:

Let: $z=4+3i$

$$|z|=\sqrt{(4)^2+(3)^2}$$
$$|z|=\sqrt{16+9}$$
$$|z|=\sqrt{25}$$
$$|z|=5$$

(ii) $-5-4i$ 10301096

Solution:

Let: $z=-5-4i$

$$|z|=\sqrt{(-5)^2+(-4)^2}$$
$$|z|=\sqrt{25+16}$$
$$|z|=\sqrt{41}$$

(iii) $\frac{3}{5}-\frac{4}{5}i$ 10301097

Solution:

Let: $z=\frac{3}{5}-\frac{4}{5}i$

$$|z|=\sqrt{\left(\frac{3}{5}\right)^2+\left(\frac{-4}{5}\right)^2}$$
$$|z|=\sqrt{\frac{9}{25}+\frac{16}{25}}$$
$$|z|=\sqrt{\frac{25}{25}}$$
$$|z|=\sqrt{1}$$
$$|z|=1$$

(iv) $-\sqrt{2}-\sqrt{3}\,i$ 10301098

Solution:

Let: $z=-\sqrt{2}-\sqrt{3}i$

$$|z|=\sqrt{\left(-\sqrt{2}\right)^2+\left(-\sqrt{3}\right)^2}$$
$$|z|=\sqrt{2+3}$$
$$|z|=\sqrt{5}$$

2. If $z_1=2+7i$ and $z_2=4-3i$, then verify that

(i) $\overline{z_1+z_2}=\overline{z_1}+\overline{z_2}$ 10301099

Solution:

Given that: $z_1=2+7i,\ z_2=4-3i$

L.H.S $=\overline{z_1+z_2}$

$$z_1+z_2=(2+7i)+(4-3i)$$
$$=2+7i+4-3i$$
$$z_1+z_2=6+4i$$

Taking conjugate on both sides, we have

$$\overline{z_1+z_2}=6-4i \quad \ldots\ldots\ldots\text{(i)}$$

R.H.S $=\overline{z_1}+\overline{z_2}$

$$z_1=2+7i \Rightarrow \overline{z_1}=2-7i$$
$$z_2=4-3i \Rightarrow \overline{z_2}=4+3i$$
$$\overline{z_1}+\overline{z_2}=(2-7i)+(4+3i)$$
$$=2-7i+4+3i$$
$$\overline{z_1}+\overline{z_2}=6-4i \quad \ldots\ldots\ldots\text{(ii)}$$

From equation (i) and (ii), we have

L.H.S. = R.H.S.

$\overline{z_1+z_2}=\overline{z_1}+\overline{z_2}$ Hence proved.

(ii) $\overline{z_1\cdot z_2}=\overline{z_1}\cdot\overline{z_2}$ 10301100

Solution:

Given that: $z_1=2+7i, z_2=4-3i$

L.H.S $=\overline{z_1\cdot z_2}$

$$z_1\cdot z_2=(2+7i)(4-3i)$$
$$=2(4-3i)+7i(4-3i)$$
$$=8-6i+28i-21i^2$$
$$=8+22i-21(-1) \quad \boxed{\because i^2=-1}$$
$$=8+22i+21$$
$$z_1\cdot z_2=29+22i$$

Taking conjugate on both sides, we have

$$\overline{z_1\cdot z_2}=29-22i \quad \ldots\ldots\ldots\text{(i)}$$

R.H.S $= \overline{z_1} \cdot \overline{z_2}$

$z_1 = 2+7i \Rightarrow \overline{z_1} = 2-7i$

$z_2 = 4-3i \Rightarrow \overline{z_2} = 4+3i$

Now,

$$\begin{aligned}\overline{z_1} \cdot \overline{z_2} &= (2-7i)(4+3i)\\ &= 2(4+3i) - 7i(4+3i)\\ &= 8+6i-28i-21i^2\\ &= 8-22i-21(-1) \qquad \boxed{\because i^2=-1}\\ &= 8-22i+21\end{aligned}$$

$\overline{z_1} \cdot \overline{z_2} = 29-22i$

From equation (i) and (ii), we have
L.H.S. = R.H.S.

$\overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$. Hence proved.

(iii) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$ **10301101**

Solution:

Given that: $z_1 = 2+7i, z_2 = 4-3i$

L.H.S $= \overline{\left(\frac{z_1}{z_2}\right)}$

$$\begin{aligned}\frac{z_1}{z_2} &= \frac{2+7i}{4-3i}\\ &= \frac{2+7i}{4-3i} \times \frac{4+3i}{4+3i}\\ &= \frac{2(4+3i)+7i(4+3i)}{(4-3i)(4+3i)}\\ &= \frac{8+6i+28i+21i^2}{(4)^2-(3i)^2}\end{aligned}$$

$\boxed{\because (a+b)(a-b) = a^2-b^2}$

$$\begin{aligned}&= \frac{8+34i+21i^2}{16-9i^2}\\ &= \frac{8+34i+21(-1)}{16-9(-1)} \qquad \boxed{\because i^2=-1}\\ &= \frac{8+34i-21}{16+9}\end{aligned}$$

$\frac{z_1}{z_2} = \frac{-13+34i}{25}$

$\frac{z_1}{z_2} = \frac{-13}{25} + \frac{34}{25}i$

Taking conjugate on both sides, we have

$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{-13}{25} - \frac{34}{25}i$ ………(i)

R.H.S $= \frac{\overline{z_1}}{\overline{z_2}}$

Given that:

$z_1 = 2+7i \Rightarrow \overline{z_1} = 2-7i$

$z_2 = 4-3i \Rightarrow \overline{z_2} = 4+3i$

Now,

$$\begin{aligned}\frac{\overline{z_1}}{\overline{z_2}} &= \frac{2-7i}{4+3i}\\ &= \frac{2-7i}{4+3i} \times \frac{4-3i}{4-3i}\\ &= \frac{2(4-3i)-7i(4-3i)}{(4+3i)(4-3i)}\\ &= \frac{8-6i-28i+21i^2}{(4)^2-(3i)^2}\end{aligned}$$

$\boxed{\because (a+b)(a-b) = a^2-b^2}$

$$\begin{aligned}&= \frac{8-34i+21i^2}{16-9i^2}\\ &= \frac{8-34i+21(-1)}{16-9(-1)} \qquad \boxed{\because i^2=-1}\\ &= \frac{8-34i-21}{16+9}\end{aligned}$$

$\frac{\overline{z_1}}{\overline{z_2}} = \frac{-13-34i}{25}$

$\frac{\overline{z_1}}{\overline{z_2}} = \frac{-13}{25} - \frac{34}{25}i$ ………(ii)

From equation (i) and (ii), we have
L.H.S. = R.H.S.

$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$. Hence proved.

3. If $z = 5-2i$, then verify that

(i) $\bar{\bar{z}} = z$ **10301102**

Solution:

Given that: $z = 5-2i$

L.H.S $= \bar{\bar{z}}$

Now, $z = 5-2i$

Taking conjugate on both sides, we have

$\bar{z} = 5+2i$

Again taking conjugate, we have

$\bar{\bar{z}} = 5-2i$

$= z =$ R.H.S.

Hence proved.

(ii) $|z| = |\bar{z}|$ **10301103**

Solution:

Given that: $z = 5-2i$

L.H.S $= |z|$

Now, $z = 5-2i$

$|z| = \sqrt{(5)^2 + (-2)^2}$

$= \sqrt{25+4}$

$|z| = \sqrt{29}$ ....(i)

R.H.S $= |\bar{z}|$

Given: $z = 5-2i \Rightarrow \bar{z} = 5+2i$

$|\bar{z}| = \sqrt{(5)^2 + (2)^2}$

$= \sqrt{25+4}$

$|\bar{z}| = \sqrt{29}$ ....(ii)

From equation (i) and (ii), we have

L.H.S. = R.H.S.

$|z| = |\bar{z}|$. Hence proved.

(iii) $|z| = |-z|$ **10301104**

Solution:

L.H.S $= |z|$

Given that: $z = 5-2i$

$|z| = \sqrt{(5)^2 + (-2)^2}$

$|z| = \sqrt{25+4}$

$|z| = \sqrt{29}$ ....(i)

R.H.S $= |-z|$

$z = 5-2i \Rightarrow -z = -5+2i$

$|-z| = \sqrt{(-5)^2 + (2)^2}$

$|-z| = \sqrt{25+4}$

$|-z| = \sqrt{29}$ ....(ii)

From equation (i) and (ii), we have

L.H.S. = R.H.S.

$|z| = |-z|$. Hence proved.

(iv) $z\bar{z} = |z|^2$ **10301105**

Solution:

L.H.S $= z\bar{z}$

Given that: $z = 5-2i \Rightarrow \bar{z} = 5+2i$

$z\bar{z} = (5-2i)(5+2i)$

$= (5)^2 - (2i)^2$ $\boxed{\because (a-b)(a+b) = a^2 - b^2}$

$= 25 - 4i^2$

$= 25 - 4(-1)$ $\boxed{\because i^2 = -1}$

$= 25+4$

$z\bar{z} = 29$ ....(i)

R.H.S $= |z|^2$

Given that: $z = 5-2i$

$|z| = \sqrt{(5)^2 + (-2)^2}$

$= \sqrt{25+4}$

$|z| = \sqrt{29}$

Taking square on both sides, we have

$|z|^2 = \left(\sqrt{29}\right)^2$

$|z|^2 = 29$ ....(ii)

From equation (i) and (ii), we have

L.H.S. = R.H.S.

$z \cdot \bar{z} = |z|^2$. Hence proved.

(v) $|z| = |-\bar{z}|$ **10301106**

Solution:

L.H.S $= |z|$

Given that: $z = 5 - 2i$

$|z| = \sqrt{(5)^2 + (-2)^2}$

$|z| = \sqrt{25+4}$

$|z| = \sqrt{29}$ ………(i)

R.H.S $= |-\bar{z}|$

Given that: $z = 5 - 2i$

Now, $\bar{z} = 5 + 2i$

and $-\bar{z} = -5 - 2i$

$|-\bar{z}| = \sqrt{(-5^2) + (-2)^2}$

$= \sqrt{25+4}$

$|-\bar{z}| = \sqrt{29}$ ………(ii)

From equation (i) and (ii), we have

L.H.S. = R.H.S.

$|z| = |-\bar{z}|^2$ Hence proved.

4. If $z = 4 - 3i$, then verify that $|z| = |-z| = |\bar{\bar{z}}| = |-\bar{z}|$ **10301107**

Solution:

Given that:

$z = 4 - 3i$

$|z| = \sqrt{(4)^2 + (-3)^2}$

$|z| = \sqrt{16+9}$

$|z| = \sqrt{25}$

$|z| = 5$ ....(i)

Now,

$z = 4 - 3i$

and $-z = -4 + 3i$

$|-z| = \sqrt{(-4)^2 + (3)^2}$

$|-z| = \sqrt{16+9}$

$|-z| = \sqrt{25}$

$|-z| = 5$ ………(ii)

Also,

$z = 4 - 3i$

$\Rightarrow \bar{z} = 4 + 3i$

$\Rightarrow \bar{\bar{z}} = 4 - 3i$

$|\bar{\bar{z}}| = \sqrt{(4)^2 + (-3)^2}$

$= \sqrt{16+9}$

$= \sqrt{25}$

$|\bar{\bar{z}}| = 5$ ………(iii)

Also,

$z = 4 - 3i$

$-z = -4 + 3i$

$-\bar{z} = -4 - 3i$

$|-\bar{z}| = \sqrt{(-4)^2 + (-3)^2}$

$= \sqrt{16+9}$

$= \sqrt{25}$

$|-\bar{z}| = 5$ ………(iv)

From equation (i), (ii), (iii) and (iv), we have

$|z| = |-z| = |\bar{\bar{z}}| = |-\bar{z}|$

5. If $z_1 = 2 + 3i, z_2 = -1 + i$, evaluate:

Solution: **10301108**

Given that:

$z_1 = 2 + 3i$

$z_2 = -1 + i$

$z_1 z_2 = (2+3i)(-1+i)$

$= 2(-1+i) + 3i(-1+i)$

$= -2 + 2i - 3i + 3i^2$

$= -2 - i + 3(-1)$ $\because i^2 = -1$

$= -2 - i - 3$

$z_1 z_2 = -5 - i$

(i) $\text{Re}(z_1 z_2)$

Taking real part, we have

$\text{Re}(z_1 z_2) = -5$

(ii) $\text{Im}(z_1 z_2)$

Taking imaginary part, we have

$\text{Img}(z_1 z_2) = -1$

Finding Real and Imaginary Parts of Complex Numbers

Let $z=(x+iy)^n$ be a complex number.

(i) For $n=1$

We have $z=(x+iy)^1$

$$=x+iy$$

$$\text{Re}(z)=x \text{ and } \text{Im}(z)=y$$

(ii) For $n=-1$

We have $z=(x+iy)^{-1}$

$$z=\frac{1}{x+iy}$$

$$=\frac{x-iy}{(x+iy)(x-iy)}$$

$$=\frac{x-iy}{x^2+y^2}$$

$$=\frac{x}{x^2+y^2}-\frac{iy}{x^2+y^2}$$

$$\therefore\ \text{Re}(z)=\frac{x}{x^2+y^2}, \text{Im}(z)=\frac{-y}{x^2+y^2}$$

Skilled Practice

Q. Find real and imaginary parts of $(x+iy)^n$ for $n=\pm 2$ **10301109**

Solution:

Let $z=(x+iy)^n$

When $n=2$

$$z=(x+iy)^2$$

$$z=(x)^2+(iy)^2+2(x)(iy)$$

$$\because (a+b)^2=a^2+b^2+2ab$$

$$z=x^2+i^2y^2+2xyi$$

$$z=x^2+(-1)y^2+2xyi \qquad \because i^2=-1$$

$$z=x^2-y^2+2xyi$$

$$\text{Re}(z)=x^2-y^2, \qquad Img(z)=2xy$$

Now, when $n=-2$

$$z=(x+iy)^{-2}$$

$$z=\frac{1}{(x+iy)^2} \qquad \because a^{-2}=\frac{1}{a^2}$$

$$z=\frac{1}{(x)^2+(iy)^2+2(x)(iy)}$$

$$\because (a+b)^2=a^2+b^2+2ab$$

$$z=\frac{1}{x^2+i^2y^2+2xyi}$$

$$z=\frac{1}{x^2+(-1)y^2+2xyi} \qquad \because i^2=-1$$

$$z=\frac{1}{x^2-y^2+2xyi}$$

$$z=\frac{1}{(x^2-y^2)+2xyi}\times\frac{(x^2-y^2)-2xyi}{(x^2-y^2)-2xyi}$$

$$z=\frac{(x^2-y^2)-2xyi}{(x^2-y^2)^2-(2xyi)^2}$$

$$z=\frac{(x^2-y^2)-2xyi}{x^4+y^4-2x^2y^2-4x^2y^2i^2}$$

$$z=\frac{(x^2-y^2)-2xyi}{x^4+y^4-2x^2y^2-4x^2y^2(-1)}$$

$$z=\frac{(x^2-y^2)-2xyi}{x^4+y^4-2x^2y^2+4x^2y^2}$$

$$z=\frac{(x^2-y^2)-2xyi}{x^4+y^4+2x^2y^2}$$

$$z=\frac{(x^2-y^2)-2xyi}{(x^2+y^2)^2}$$

$$z=\frac{(x^2-y^2)}{(x^2+y^2)^2}-\frac{2xy}{(x^2+y^2)^2}i$$

So,

$$\text{Re}(z)=\frac{x^2-y^2}{(x^2+y^2)^2}, \quad Img(z)=\frac{-2xy}{(x^2+y^2)^2}$$

Example 17: **Find real and imaginary parts of $(4+3i)^{-1}$.** **10301110**

Solution:

Let $z=(4+3i)^{-1}$

$$=\frac{1}{(4+3i)}$$

$$=\frac{1}{(4+3i)}\times\frac{4-3i}{4-3i}$$

$$=\frac{1(4-3i)}{(4+3i)(4-3i)}$$

$$=\frac{4-3i}{(4)^2-(3i)^2}$$

$$=\frac{4-3i}{16-9i^2}$$

$$=\frac{4-3i}{16-9(-1)}$$

$$=\frac{4-3i}{16+9}$$

$$=\frac{4-3i}{25}$$

$$z=\frac{4}{25}-\frac{3}{25}i$$

$\therefore$ $\text{Re}(z)=\frac{4}{25}$ and $\text{Im}(z)=\frac{-3}{25}$

Example 18: **Find real and imaginary parts of $z=(4+3i)^{-2}$.** **10301111**

Solution:

Let $z=(4+3i)^{-2}$

$$=\frac{1}{(4+3i)^2}$$

$$=\frac{1}{(4)^2+(3i)^2+2(4)(3i)}$$

$$=\frac{1}{16+9i^2+24i}$$

$$=\frac{1}{16-9+24i}$$

$$=\frac{1}{7+24i}$$

$$=\frac{1}{7+24i}\times\frac{7-24i}{7-24i}$$

$$=\frac{(7-24i)}{(7+24i)(7-24i)}$$

$$=\frac{7-24i}{(7)^2-(24i)^2}$$

$$=\frac{7-24i}{49-576i^2}$$

$$=\frac{7-24i}{49-(-576)}$$

$$=\frac{7-24i}{49+576}$$

$$=\frac{7-24i}{625}$$

$$=\frac{7}{625}-\frac{24}{625}i$$

$\therefore$ $\text{Re}(z)=\frac{7}{625}, \text{Im}(z)=\frac{-24}{625}$

> **Do you know?**
>
> $\text{Re}(z)=\frac{z+\overline{z}}{2}$, $\text{Im}(z)=\frac{z-\overline{z}}{2i}$

Example 19: **Find real and imaginary parts of $z=\left(\frac{4+3i}{3+2i}\right)^{-1}$.** **10301112**

Solution:

$$z=\left(\frac{4+3i}{3+2i}\right)^{-1}$$

$$=\frac{3+2i}{4+3i} \qquad \left[\because \left(\frac{a}{b}\right)^{-m}=\left(\frac{b}{a}\right)^{m}\right]$$

$$=\frac{(3+2i)(4-3i)}{(4+3i)(4-3i)}$$

$$=\frac{12-9i+8i-6i^2}{(4)^2-(3i)^2}$$

$$=\frac{12-i-6(-1)}{16-9i^2} \qquad \left[\because \ ^2 \quad 1\right]$$

$$=\frac{12-i+6}{16+9}$$

$$=\frac{18-i}{25}=\frac{18}{25}-\frac{1}{25}i$$

$$\therefore \quad \text{Re}(z)=\frac{18}{25}, \text{Im}(z)=\frac{-1}{25}$$

Example 20: **Find real and imaginary parts of $z=\left(\frac{1+i}{1-i}\right)^{-2}$.** 10301113

Solution:

$$z=\left(\frac{1+i}{1-i}\right)^{-2}$$

$$=\left(\frac{1-i}{1+i}\right)^{2} \qquad \boxed{\because \left(\frac{a}{b}\right)^{-m}=\left(\frac{b}{a}\right)^{m}}$$

$$=\left(\frac{1-i}{1+i}\times\frac{1-i}{1-i}\right)^{2}$$

$$=\left(\frac{1-i-i+i^2}{1^2-i^2}\right)^{2}$$

$$=\left(\frac{1-2i-1}{1+1}\right)^{2}=\left(\frac{-2i}{2}\right)^{2}=(-i)^2=i^2=-1$$

$$\therefore \quad \text{Re}(z)=-1, \text{Im}(z)=0$$

Solution of Simultaneous Linear Equations with Complex Coefficient

Q. Define simultaneous equations. 10301114

Ans. More than one equation which are to be satisfied by the same values of the variables involved are called simultaneous equations or a system of equations.

Example 21: **Solve the given simultaneous linear equations with complex coefficients for z and w:** 10301115

$$5z-(3+i)w=7-i$$

$$(2-i)z+2iw=4$$

Solution:

$$5z-(3+i)w=7-i \qquad \text{.........(i)}$$

$$(2-i)z+2iw=4 \qquad \text{.........(ii)}$$

Step I:

From equation (i), we solve for w in terms of z.

$$5z-(3+i)w=7-i$$

$$5z-7+i=(3+i)w$$

$$\Rightarrow \quad (3+i)w=5z-7+i$$

$$w=\frac{5z-7+i}{3+i} \qquad \text{.........(iii)}$$

Step II:

Put the expression of w in equation (ii)

$$(2-i)z+2i\left(\frac{5z-7+i}{3+i}\right)=4$$

Multiplying both sides by $(3+i)$, we have

$$(2-i)(3+i)z+2i(5z-7+i)=4(3+i)$$

$$(6+2i-3i-i^2)z+10iz-14i+2i^2=12+4i$$

$$(6-i+1)z+10iz-14i-2=12+4i$$

$$(7-i)z+10iz-14i-2=12+4i$$

$$7z-iz+10iz-14i-2=12+4i$$

$$7z+9iz=12+4i+14i+2$$

$$7z+9iz=14+18i$$

$$z(7+9i)=2(7+9i)$$

$$z=\frac{2(7+9i)}{7+9i}$$

$$z=2$$

Step III:

Put $z=2$ in (iii), we get

$$w=\frac{5(2)-7+i}{3+i}$$

$$w=\frac{10-7+i}{3+i}$$

$$w=\frac{3+i}{3+i}=1$$

Hence, $z=2, w=1$

EXERCISE 1.4

1. Find the real and imaginary parts of the following complex numbers:

(i) $(8-3i)^2$ 10301116

Solution:

Let $z=(8-3i)^2$

$z=(8)^2+(3i)^2-2(8)(3i)$

$\because (a-b)^2=a^2+b^2-2ab$

$z=64+9i^2-48i$

$z=64+9(-1)-48i$ $\quad \because i^2=-1$

$z=64-9-48i$

$z=55-48i$

Real $(z)=55$, Img $(z)=-48$

(ii) $(5+3i)^{-1}$ 10301117

Solution:

Let $z=(5+3i)^{-1}$

$z=\dfrac{1}{(5+3i)}$

$z=\dfrac{1}{5+3i}\times\dfrac{5-3i}{5-3i}$

$z=\dfrac{5-3i}{(5)^2-(3i)^2}$ $\quad \because (a+b)(a-b)=a^2-b^2$

$z=\dfrac{5-3i}{25-9i^2}$

$z=\dfrac{5-3i}{25-9(-1)}$ $\quad \because i^2=-1$

$z=\dfrac{5-3i}{25+9}$

$z=\dfrac{5-3i}{34}$

$z=\dfrac{5}{34}-\dfrac{3}{34}i$

Re $(z)=\dfrac{5}{34}$, Img $(z)=\dfrac{-3}{34}$

(iii) $(4-5i)^{-1}$ 10301118

Solution:

Let $z=(4-5i)^{-1}$

$z=\dfrac{1}{4-5i}$

$z=\dfrac{1}{4-5i}\times\dfrac{4+5i}{4+5i}$

$z=\dfrac{4+5i}{(4-5i)(4+5i)}$

$z=\dfrac{4+5i}{(4)^2-(5i)^2}$ $\quad \because a^2-b^2=(a+b)(a-b)$

$z=\dfrac{4+5i}{16-25i^2}$

$z=\dfrac{4+5i}{16-25(-1)}$ $\quad \because i^2=-1$

$z=\dfrac{4+5i}{16+25}$

$z=\dfrac{4+5i}{41}$

$z=\dfrac{4}{41}+\dfrac{5}{41}i$

Re $(z)=\dfrac{4}{41}$, Img $(z)=\dfrac{5}{41}$

(iv) $(4-3i)^{-2}$ 10301119

Solution:

Let $z=(4-3i)^{-2}$

$z=\dfrac{1}{(4-3i)^2}$

$z=\dfrac{1}{(4)^2+(3i)^2-2(4)(3i)}$

$\because (a-b)^2=a^2+b^2-2ab$

$z=\dfrac{1}{16+9i^2-24i}$

$z=\dfrac{1}{16+9(-1)-24i}$ $\quad \because i^2=-1$

$$z=\frac{1}{16-9-24i}$$

$$z=\frac{1}{7-24i}$$

$$z=\frac{1}{7-24i}\times\frac{7+24i}{7+24i}$$

$$z=\frac{7+24i}{(7)^2-(24i)^2}\quad \boxed{\because (a+b)(a-b)=a^2-b^2}$$

$$z=\frac{7+24i}{49-576i^2}$$

$$z=\frac{7+24i}{49-576(-1)}\quad \boxed{\because i^2=-1}$$

$$z=\frac{7+24i}{49+576}$$

$$z=\frac{7+24i}{625}$$

$$z=\frac{7}{625}+\frac{24}{625}i$$

$$\text{Re }(z)=\frac{7}{625},\quad \text{Img }(z)=\frac{24}{625}$$

(v) $\left(\frac{3+2i}{4+3i}\right)^{-1}$ **10301120**

Solution:

$$\text{Let }\ z=\left[\frac{3+2i}{4+3i}\right]^{-1}$$

$$z=\left[\frac{4+3i}{3+2i}\right]^{1}\quad \boxed{\because \left(\frac{a}{b}\right)^{-1}=\left(\frac{b}{a}\right)^{1}}$$

$$z=\frac{4+3i}{3+2i}$$

$$z=\frac{4+3i}{3+2i}\times\frac{3-2i}{3-2i}$$

$$z=\frac{(4+3i)(3-2i)}{(3+2i)(3-2i)}$$

$$z=\frac{4(3-2i)+3i(3-2i)}{(3)^2-(2i)^2}$$

$$\boxed{\because a^2-b^2=(a+b)(a-b)}$$

$$z=\frac{12-8i+9i-6i^2}{9-4i^2}$$

$$z=\frac{12+i-6(-1)}{9-4(-1)}\quad \boxed{\because i^2=-1}$$

$$z=\frac{12+i+6}{9+4}$$

$$z=\frac{18+i}{13}$$

$$z=\frac{18}{13}+\frac{1}{13}i$$

$$\text{Re }(z)=\frac{18}{13},\quad \text{Img }(z)=\frac{1}{13}$$

(vi) $\left(\frac{2-i}{2+i}\right)^{-2}$ **10301121**

Solution:

$$\text{Let }\ z=\left[\frac{2-i}{2+i}\right]^{-2}$$

$$z=\left(\frac{2+i}{2-i}\right)^{2}\quad \boxed{\because \left(\frac{a}{b}\right)^{-2}=\left(\frac{b}{a}\right)^{2}}$$

$$z=\frac{(2+i)^2}{(2-i)^2}$$

$$z=\frac{2^2+i^2+2(2)(i)}{2^2+i^2-2(2)(i)}$$

$$z=\frac{4+(-1)+4i}{4+(-1)-4i}\quad \boxed{\because i^2=-1}$$

$$z=\frac{4-1+4i}{4-1-4i}$$

$$z=\frac{3+4i}{3-4i}$$

$$z=\frac{3+4i}{3-4i}\times\frac{3+4i}{3+4i}$$

$$z=\frac{9+12i+12i+16i^2}{9+12i-12i-16i^2}$$

$$z=\frac{9+24i+16(-1)}{9-16(-1)} \quad \boxed{\because i^2=-1}$$

$$z=\frac{9+24i-16}{9+16}$$

$$z=\frac{-7+24i}{25}$$

$$z=\frac{-7}{25}+\frac{24}{25}i$$

$$\text{Re}\,(z)=\frac{-7}{25},\quad \text{Img}\,(z)=\frac{24}{25}$$

(vii) $\left(\frac{1-2i}{1+i}\right)^2$ **10301122**

Solution:

Let $z=\left(\frac{1-2i}{1+i}\right)^2$

$$z=\frac{(1-2i)^2}{(1+i)^2}$$

$$z=\frac{(1)^2+(2i)^2-2(1)(2i)}{(1)^2+(i)^2+2(1)(i)}$$

$$z=\frac{1+4i^2-4i}{1+i^2+2i}$$

$$z=\frac{1+4(-1)-4i}{1+(-1)+2i} \quad \boxed{\because i^2=-1}$$

$$z=\frac{1-4-4i}{1-1+2i}$$

$$z=\frac{-3-4i}{2i}\times\frac{-2i}{-2i}$$

$$z=\frac{6i+8i^2}{-4i^2}$$

$$z=\frac{6i+8(-1)}{-4(-1)}$$

$$z=\frac{6i-8}{4}$$

$$z=\frac{6i}{4}-\frac{8}{4}$$

$$z=\frac{3}{2}i-2$$

$$z=-2+\frac{3}{2}i$$

$$\text{Re}\,(z)=-2,\ \text{Img}\,(z)=\frac{3}{2}$$

2. Solve the following simultaneous linear equations with complex coefficients for w and z:

(i) $3z+(2+i)w=11-i$ **10301123**

$(2-i)z-w=-1+i$

Solution:

$3z+(2+i)w=11-i$ ....(i)

$(2-i)z-w=-1+i$ ....(ii)

From equation (i)

$$3z+(2+i)w=11-i$$

$$(2+i)w=11-i-3z$$

$$w=\frac{11-i-3z}{2+i} \quad ....(iii)$$

Put equation (iii) in equation (ii), we have:

$$(2-i)z-\left[\frac{11-i-3z}{2+i}\right]=-1+i$$

Multiplying both sides by $(2+i)$

$$(2+i)(2-i)z-(11-i-3z)=(2+i)(-1+i)$$

$$\left[(2)^2-(i)^2\right]z-(11-i-3z)=2(-1+i)+i(-1+i)$$

$$(4-i^2)z-11+i+3z=-2+2i-i+i^2$$

$$\left[4-(-1)\right]z-11+i+3z=-2+i+(-1)$$

$$\boxed{\because i^2=-1}$$

$$(4+1)z-11+i+3z=-2+i-1$$

$$5z-11+i+3z=-3+i$$

$$8z-11+i=-3+i$$

$$8z=-3+11+i-i$$

$$8z=8$$

$z=\frac{8}{8}$

$z=1$

Put $z=1$ in equation (iii), we have

$w=\frac{11-i-3(1)}{2+i}$

$w=\frac{8-i}{2+i}$

$w=\frac{8-i}{2+i}\times\frac{2-i}{2-i}$

$w=\frac{8(2-i)-i(2-i)}{(2)^2-(i)^2}$

$w=\frac{16-8i-2i+i^2}{4-i^2}$

$\boxed{\because a^2-b^2=(a+b)(a-b)}$

$w=\frac{16-10i+(-1)}{4-(-1)}$ $\boxed{\because i^2=-1}$

$w=\frac{16-10i-1}{4+1}$

$w=\frac{15-10i}{5}$

$w=\frac{15}{5}-\frac{10}{5}i$

$w=3-2i$

So, $\boxed{z=1, w=3-2i}$

(ii) $2z+(3+i)w=9-i$ **10301124**

$-iz-iw=-1+i$

Solution:

$2z+(3+i)w=9-i$ ....(i)

$-iz-iw=-1+i$ ....(ii)

From equation (ii)

$-i(z+w)=-1+i$

$(z+w)=\frac{-1+i}{-i}$

$z+w=\frac{-1+i}{-i}\times\frac{i}{i}$

$z+w=\frac{i(-1+i)}{-i^2}$

$z+w=\frac{-i+i^2}{-i^2}$

$z+w=\frac{-i+(-1)}{-(-1)}$ $\boxed{\because i^2=-1}$

$z+w=\frac{-i-1}{1}$

$z+w=-1-i$

$z=-1-i-w$ ....(iii)

Put equation (iii) in equation (i)

$2(-1-i-w)+(3+i)w=9-i$

$-2-2i-2w+(3+i)w=9-i$

$-2-2i+(-2+3+i)w=9-i$

$(1+i)w=9-i+2+2i$

$(1+i)w=11+i$

$w=\frac{11+i}{1+i}$

$w=\frac{11+i}{1+i}\times\frac{1-i}{1-i}$

$w=\frac{11(1-i)+i(1-i)}{(1)^2-(i)^2}$

$\boxed{\because a^2-b^2=(a+b)(a-b)}$

$w=\frac{11-11i+i-i^2}{1-i^2}$

$w=\frac{11-10i-(-1)}{1-(-1)}$ $\boxed{\because i^2=-1}$

$w=\frac{11-10i+1}{2}$

$w=\frac{12-10i}{2}$

$w=\frac{12}{2}-\frac{10i}{2}$

$w=6-5i$

Put $w = 6 - 5i$ in equation (iii)

$z = -1 - i - (6 - 5i)$

$z = -1 - i - 6 + 5i$

$z = -7 + 4i$

So, $\boxed{z = -7 + 4i,\ w = 6 - 5i}$

(iii) $z - 4w = 3i$ **10301125**

$2z + 3w = 11 - 5i$

Solution:

$z - 4w = 3i$ ....(i)

$2z + 3w = 11 - 5i$ ....(ii)

From equation (i)

$z - 4w = 3i$

$z = 3i + 4w$ ....(iii)

Put equation (iii) in equation (ii), we have

$2(3i + 4w) + 3w = 11 - 5i$

$6i + 8w + 3w = 11 - 5i$

$6i + 11w = 11 - 5i$

$11w = 11 - 5i - 6i$

$11w = 11 - 11i$

$11w = 11(1 - i)$

$w = \dfrac{11(1-i)}{11}$

$w = 1 - i$

Put $w = 1 - i$ in equation (iii), we have

$z = 3i + 4(1 - i)$

$z = 3i + 4 - 4i$

$z = 4 - i$

So, $\boxed{w = 1 - i,\ z = 4 - i}$

(iv) $z + w = 3i$ **10301126**

$2z + 3w = 2$

Solution:

$z + w = 3i$ ...(i)

$2z + 3w = 2$ ...(ii)

From equation (i), we have

$z + w = 3i$

$z = 3i - w$ ...(iii)

Put equation (iii) in equation (ii), we have

$2(3i - w) + 3w = 2$

$6i - 2w + 3w = 2$

$w = 2 - 6i$

Put $w = 2 - 6i$ in equation (iii), we have

$z = 3i - (2 - 6i)$

$z = 3i - 2 + 6i$

$z = -2 + 9i$

So, $\boxed{w = 2 - 6i,\ z = -2 + 9i}$

(v) $2z + (3 + i)w = 1$ **10301127**

$-z - (1 - i)w = 2$

Solution:

$2z + (3 + i)w = 1$ ....(i)

$-z - (1 - i)w = 2$ ....(ii)

From equation (ii), we have

$-z = 2 + (1 - i)w$

$z = -2 - (1 - i)w$ ....(iii)

Put equation (iii) in equation (i), we have

$2[-2 - (1 - i)w] + (3 + i)w = 1$

$-4 - (2 - 2i)w + (3 + i)w = 1$

$(3 + i)w - (2 - 2i)w = 1 + 4$

$(3 + i - 2 + 2i)w = 5$

$(1 + 3i)w = 5$

$w = \dfrac{5}{1 + 3i}$

$w = \dfrac{5}{(1 + 3i)} \times \dfrac{(1 - 3i)}{(1 - 3i)}$

$w = \dfrac{5 - 15i}{(1)^2 - (3i)^2}$ $\boxed{\because a^2 - b^2 = (a + b)(a - b)}$

$w = \dfrac{5 - 15i}{1 - 9i^2}$

$w = \dfrac{5 - 15i}{1 - 9(-1)}$ $\boxed{\because i^2 = -1}$

$w = \dfrac{5 - 15i}{1 + 9}$

$w = \dfrac{5 - 15i}{10}$

$w = \dfrac{5}{10} - \dfrac{15}{10}i$

$w = \dfrac{1}{2} - \dfrac{3}{2}i$

Put $w=\frac{1}{2}-\frac{3}{2}i$ in equation (iii)

$$z=-2-(1-i)\left(\frac{1}{2}-\frac{3}{2}i\right)$$

$$z=-2-(1-i)\left(\frac{1-3i}{2}\right)$$

$$z=-2-\frac{(1-i)(1-3i)}{2}$$

$$z=-2-\frac{(1-3i-i+3i^2)}{2}$$

$$z=-2-\frac{(1-4i+3(-1))}{2} \qquad \boxed{\because i^2=-1}$$

$$z=-2-\frac{(1-4i-3)}{2}$$

$$z=-2-\frac{(-2-4i)}{2}$$

$$z=\frac{-4-(-2-4i)}{2}$$

$$z=\frac{-4+2+4i}{2}$$

$$z=\frac{-2+4i}{2}$$

$$z=\frac{-2}{2}+\frac{4}{2}i$$

$$z=-1+2i$$

So, $w=\frac{1}{2}-\frac{3}{2}i$ and $z=-1+2i$

REVIEW EXERCISE 1

1. Four possible options are given for the following questions. Choose the correct option.

(i) $i^2+i^4=$ ______ **10301128**

(a) -1 (b) 0
(c) 1 (d) 2

(ii) Real part of $(2-3i)(2+3i)$ is: **10301129**

(a) -3 (b) 1
(c) 4 (d) 13

(iii) Imaginary part of $(2-i)(2+i)$ is: **10301130**

(a) 0 (b) 1
(c) 7 (d) 9

(iv) $(x+iy)$ will be pure imaginary number, when: **10301131**

(a) $y=0$ (b) $x=0$
(c) $i=0$ (d) $x=0, y=0$

(v) What is additive inverse of $5-2i$? **10301132**

(a) $5+2i$ (b) $-5-2i$
(c) $5-2i$ (d) $-5+2i$

(vi) What is multiplicative inverse of $z=1+i$? **10301133**

(a) $1-i$ (b) i
(c) $\frac{1}{2}-\frac{1}{2}i$ (d) $\frac{-1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i$

(vii) If $z=4-3i$, then $z\bar{z}=$ ______ **10301134**

(a) 3 (b) 9
(c) 16 (d) 25

(viii) Conjugate of $9-4i$ is: **10301135**

(a) $-9-4i$ (b) $9+4i$
(c) $9+9i$ (d) $4-9i$

(ix) If $z=4+4i$, then $z+\bar{z}=$ _____ **10301136**

(a) 8 (b) $8+8i$
(c) $8i$ (d) 0

(x) If $z=5+4i$, then $|z|=$ _____ **10301137**

(a) 9 (b) 25
(c) 41 (d) $\sqrt{41}$

ANSWER KEY

(i)	b	(ii)	d	(iii)	a	(iv)	b	(v)	d
(vi)	c	(vii)	d	(viii)	b	(ix)	a	(x)	d

2. (i) Is "0" a complex number? Explain.

Solution: 10301138

Yes, '0' is a complex number.

Explanation:

A complex number is any number of the form $z = a + ib$, where a is real part, b is imaginary part and $i = \sqrt{-1}$.

Now, we consider the number zero '0'. It can be written as $0 = 0 + 0i$

Real part $= 0$, Imaginary part $= 0$

Since it fits the standard form $a + bi$, so, '0' is a complex number.

(ii) What is the result of multiplying a complex number by its conjugate?

Solution: 10301139

The result of multiplying a complex number by its conjugate is a non-negative real number.

For example: Let $z = 2 + 3i$

Its conjugate $\bar{z} = 2 - 3i$

Now, Multiplying $z \cdot \bar{z} = (2 + 3i)(2 - 3i)$

$$z \cdot \bar{z} = (2)^2 - (3i)^2$$

$$z \cdot \bar{z} = 4 - 9i^2$$

$$z \cdot \bar{z} = 4 - 9(-1) \quad \boxed{\because i^2 = -1}$$

$$z \cdot \bar{z} = 4 + 9$$

$$z \cdot \bar{z} = 13$$

Which is a non-negative real number.

(iii) State the condition for two complex numbers to be equal. 10301140

Solution:

Two complex numbers are said to be equal if and only if their real and imaginary parts are equal.

Let $z_1 = a + ib$

$z_2 = c + id$

$z_1 = z_2 \Leftrightarrow a = c$ and $b = d$

For example $z_1 = 2 + 3i$

$z_2 = 2 + 3i$

$\Rightarrow z_1 = z_2$

3. Simplify.

(i) i^{37} 10301141

Solution:

$$i^{37} = i^{36} \times i$$

$$= (i^2)^{18} \times i$$

$$= (-1)^{18} \times i \quad \boxed{\because i^2 = -1}$$

$$= 1 \times i = i$$

(ii) $i^{13} \times i^{11}$ 10301142

Solution:

$$i^{13} \times i^{11} = i^{13+11}$$

$$= i^{24} = (i^2)^{12}$$

$$= (-1)^{12} \quad \boxed{\because i^2 = -1}$$

$$= 1$$

(iii) $(-i)^{-9}$ 10301143

Solution:

$$(-i)^{-9} = \frac{1}{(-i)^9}$$

$$= \frac{1}{-i^9} = -\left[\frac{1}{i^8 \times (i)}\right]$$

$$= \left[\frac{1}{(i^2)^4 \times i}\right]$$

$$= -\left[\frac{1}{(-1)^4 \times i}\right] \quad \boxed{\because i^2 = -1}$$

$$= \frac{1}{-[1 \times i]} = \frac{1}{-i}$$

$$= \frac{1}{-i} \times \frac{i}{i}$$

$$= \frac{i}{-i^2} = \frac{i}{-(-1)} \quad \boxed{\because i^2 = -1}$$

$$= \frac{i}{1} = i$$

(iv) $(3-4i)(5-6i)$ 10301144

Solution:

$(3-4i)(5-6i)$
$=3(5-6i)-4i(5-6i)$
$=15-18i-20i+24i^2$
$=15-38i+24(-1)$ $\because i^2=-1$
$=15-38i-24$
$=-9-38i$

(v) $(3+4i)\div(5-7i)$ 10301145

Solution:

$(3+4i)\div(5-7i)$
$=\dfrac{3+4i}{5-7i}$
$=\dfrac{3+4i}{5-7i}\times\dfrac{5+7i}{5+7i}$
$=\dfrac{3(5+7i)+4i(5+7i)}{(5)^2-(7i)^2}$
$\because a^2-b^2=(a+b)(a-b)$
$=\dfrac{15+21i+20i+28i^2}{25-49i^2}$
$=\dfrac{15+41i+28(-1)}{25-49(-1)}$ $\because i^2=-1$
$=\dfrac{15+41i-28}{25+49}$
$=\dfrac{-13+41i}{74}$
$=\dfrac{-13}{74}+\dfrac{41}{74}i$

4. Find additive and multiplicative inverse of $z=8+9i$. 10301146

Solution:

Additive inverse:
Given $z=8+9i$
$-z=-8-9i$
Multiplicative Inverse:
Given: $z=8+9i$
$z^{-1}=\dfrac{1}{z}$
$z^{-1}=\dfrac{1}{8+9i}$
$z^{-1}=\dfrac{1}{8+9i}\times\dfrac{8-9i}{8-9i}$
$=\dfrac{8-9i}{(8)^2-(9i)^2}$ $\because (a+b)(a-b)=a^2-b^2$
$=\dfrac{8-9i}{64-81i^2}$
$=\dfrac{8-9i}{64-81(-1)}$ $\because i^2=-1$
$=\dfrac{8-9i}{64+81}$
$z^{-1}=\dfrac{8-9i}{145}$
$z^{-1}=\dfrac{8}{145}-\dfrac{9}{145}i$

5. If $z_1=3+4i$ and $z_2=2+3i$, then verify that

(i) $\overline{z_1+z_2}=\overline{z_1}+\overline{z_2}$ 10301147

Solution:

Given that:
$z_1=3+4i$
$z_2=2+3i$
L.H.S $=\overline{z_1+z_2}$
$z_1+z_2=(3+4i)+(2+3i)$
$z_1+z_2=3+4i+2+3i$
$z_1+z_2=5+7i$
Taking conjugate on both sides
$\overline{z_1+z_2}=5-7i$ ………(i)
R.H.S $=\overline{z_1}+\overline{z_2}$
$\overline{z_1}+\overline{z_2}=(\overline{3+4i})+(\overline{2+3i})$
$\overline{z_1}+\overline{z_2}=(3-4i)+(2-3i)$
$\overline{z_1}+\overline{z_2}=3-4i+2-3i$
$\overline{z_1}+\overline{z_2}=5-7i$ ………(ii)
From equation (i) and (ii), we have
L.H.S. = R.H.S.
$\overline{z_1+z_2}=\overline{z_1}+\overline{z_2}$. Hence proved.

(ii) $\overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$ **10301148**

Solution:

Given that:

$z_1 = 3+4i$

$z_2 = 2+3i$

$\text{L.H.S} = \overline{z_1 \cdot z_2}$

$$\begin{aligned} z_1 \cdot z_2 &= (3+4i)(2+3i) \\ &= 3(2+3i)+4i(2+3i) \\ &= 6+9i+8i+12i^2 \\ &= 6+17i+12(-1) \quad \boxed{\because i^2=-1} \\ &= 6+17i-12 \end{aligned}$$

$z_1 \cdot z_2 = -6+17i$

Taking conjugate on both sides, we get

$\overline{z_1 \cdot z_2} = -6-17i$ ....(i)

R.H.S: $= \overline{z_1} \cdot \overline{z_2}$

$\overline{z_1} \cdot \overline{z_2} = \left(\overline{3+4i}\right) \cdot \left(\overline{2+3i}\right)$

$$\begin{aligned} \overline{z_1} \cdot \overline{z_2} &= (3-4i) \cdot (2-3i) \\ &= 3(2-3i)-4i(2-3i) \\ &= 6-9i-8i+12i^2 \\ &= 6-17i+12(-1) \quad \boxed{\because i^2=-1} \\ &= 6-17i-12 \end{aligned}$$

$\overline{z_1} \cdot \overline{z_2} = -6-17i$ ....(ii)

From equation (i) and (ii), we have

L.H.S. = R.H.S.

$\overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$. Hence proved.

(iii) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$ **10301149**

Solution:

Given that:

$z_1 = 3+4i$

$z_2 = 2+3i$

$\text{L.H.S} = \overline{\left(\frac{z_1}{z_2}\right)}$

$$\frac{z_1}{z_2} = \frac{3+4i}{2+3i}$$

$$\frac{z_1}{z_2} = \frac{3+4i}{2+3i} \times \frac{2-3i}{2-3i}$$

$$= \frac{3(2-3i)+4i(2-3i)}{(2)^2-(3i)^2}$$

$$\boxed{\because a^2-b^2=(a+b)(a-b)}$$

$$= \frac{6-9i+8i-12i^2}{4-9i^2}$$

$$= \frac{6-i-12(-1)}{4-9(-1)} \quad \boxed{\because i^2=-1}$$

$$= \frac{6-i+12}{4+9}$$

$$\frac{z_1}{z_2} = \frac{18-i}{13}$$

$$\frac{z_1}{z_2} = \frac{18}{13} - \frac{1}{13}i$$

Taking conjugate on both sides, we have

$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{18}{13} + \frac{1}{13}i$ ....(i)

$\text{R.H.S} = \frac{\overline{z_1}}{\overline{z_2}}$

$$\frac{\overline{z_1}}{\overline{z_2}} = \frac{3-4i}{2-3i}$$

$$\frac{\overline{z_1}}{\overline{z_2}} = \frac{(3-4i)}{(2-3i)} \times \frac{(2+3i)}{(2+3i)}$$

$$= \frac{3(2+3i)-4i(2+3i)}{(2)^2-(3i)^2}$$

$$\boxed{\because a^2-b^2=(a+b)(a-b)}$$

$$= \frac{6+9i-8i-12i^2}{4-9i^2}$$

$$= \frac{6+i-12(-1)}{4-9(-1)} \quad \boxed{\because i^2=-1}$$

$$= \frac{6+i+12}{4+9}$$

$\dfrac{\overline{z_1}}{\overline{z_2}}=\dfrac{18+i}{13}$

$\dfrac{\overline{z_1}}{\overline{z_2}}=\dfrac{18}{13}+\dfrac{1}{13}i$ ………(ii)

From equation (i) and (ii), we have
L.H.S. = R.H.S.

$\overline{\left(\dfrac{z_1}{z_2}\right)}=\dfrac{\overline{z_1}}{\overline{z_2}}$. Hence proved.

(iv) $|z_1|=|-\overline{z_1}|$ **10301150**

Solution:

Given that:

$z_1=3+4i$

$z_2=2+3i$

$\text{L.H.S}=|z_1|$

$|z_1|=|3+4i|$

$=\sqrt{(3)^2+(4)^2}$

$=\sqrt{9+16}$

$|z_1|=\sqrt{25}$

$|z_1|=5$ ………(i)

$\text{R.H.S}=|-\overline{z_1}|$

$z_1=3+4i$

$\overline{z_1}=3-4i$

$-\overline{z_1}=-(3-4i)$

$-\overline{z_1}=-3+4i$

$|-\overline{z_1}|=|-3+4i|$

$|-\overline{z_1}|=\sqrt{(-3)^2+(4)^2}$

$|-\overline{z_1}|=\sqrt{9+16}$

$|-\overline{z_1}|=\sqrt{25}$

$|-\overline{z_1}|=5$ ………(ii)

From equation (i) and (ii), we have
L.H.S. = R.H.S.

$|z_1|=|-\overline{z_1}|$. Hence proved.

(v) $\overline{\overline{z_2}}=z_2$ **10301151**

Solution:

Given that: $z_2=2+3i$ ………(i)

$\text{L.H.S}=\overline{\overline{z_2}}$

$z_2=2+3i$

$\overline{z_2}=\overline{2+3i}$

$\overline{z_2}=2-3i$

Again taking conjugate on both sides, we have

$\overline{\overline{z_2}}=\overline{2-3i}$

$\overline{\overline{z_2}}=2+3i$ ………(ii)

$\overline{\overline{z_2}}=z_2=\text{R.H.S.}$

Hence proved.

(vi) $z_1\overline{z_1}=|z_1|^2$ **10301152**

Solution:

Given that: $z_1=3+4i$

$\Rightarrow \overline{z_1}=3-4i$

$\text{L.H.S}=z\overline{z_1}$

$=(3+4i)(3-4i)$

$z\overline{z_1}=(3)^2-(4i)^2$

$\boxed{\because (a+b)(a-b)=a^2-b^2}$

$=9-16i^2$

$=9-16(-1)$ $\boxed{\because i^2=-1}$

$=9+16$

$z\overline{z_1}=25$ ………(i)

Now,

$\text{R.H.S}=|z_1|^2$

$|z_1|=\sqrt{(3)^2+(4)^2}$

$|z_1|=\sqrt{9+16}$

$|z_1|=\sqrt{25}$

Taking square on both sides, we have

$|z_1|^2=25$ ………(ii)

From equation (i) and (ii), we have
L.H.S. = R.H.S.

$z_1\overline{z_1}=|z_1|^2$. Hence proved.

6. If $z_1 = 5+4i, z_2 = 3+2i$, then find

(i) $z_1 z_2$ **10301153**

Solution:

Given that:

$z_1 = 5+4i$

$z_2 = 3+2i$

$$z_1 z_2 = (5+4i)(3+2i)$$
$$= 5(3+2i)+4i(3+2i)$$
$$= 15+10i+12i+8i^2$$
$$= 15+22i+8(-1) \quad \boxed{\because i^2=-1}$$
$$= 15+22i-8$$
$$= 7+22i$$

(ii) $\dfrac{z_1}{z_2}$ **10301154**

Solution:

Given that:

$z_1 = 5+4i$

$z_2 = 3+2i$

$$\frac{z_1}{z_2} = \frac{5+4i}{3+2i}$$
$$\frac{z_1}{z_2} = \frac{(5+4i)}{(3+2i)} \times \frac{(3-2i)}{(3-2i)}$$
$$= \frac{5(3-2i)+4i(3-2i)}{(3)^2-(2i)^2}$$
$$\boxed{\because (a+b)(a-b) = a^2-b^2}$$
$$= \frac{15-10i+12i-8i^2}{9-4i^2}$$
$$= \frac{15+2i-8(-1)}{9-4(-1)} \quad \boxed{\because i^2=-1}$$
$$= \frac{15+2i+8}{9+4}$$
$$= \frac{23+2i}{13}$$
$$\frac{z_1}{z_2} = \frac{23}{13}+\frac{2}{13}i$$

(iii) $\overline{z_1}\cdot\overline{z_2}$ **10301155**

Solution:

Given that:

$z_1 = 5+4i$

$z_2 = 3+2i$

$$\overline{z_1}\cdot\overline{z_2} = \left(\overline{5+4i}\right)\left(\overline{3+2i}\right)$$
$$\overline{z_1}\cdot\overline{z_2} = (5-4i)(3-2i)$$
$$= 5(3-2i)-4i(3-2i)$$
$$= 15-10i-12i+8i^2$$
$$= 15-22i+8(-1) \quad \boxed{\because i^2=-1}$$
$$= 15-22i-8$$
$$= 7-22i$$

(iv) $|z_1 z_2|$ **10301156**

Solution:

Given that:

$z_1 = 5+4i$

$z_2 = 3+2i$

$$z_1 z_2 = (5+4i)(3+2i)$$
$$= 5(3+2i)+4i(3+2i)$$
$$= 15+10i+12i+8i^2$$
$$= 15+22i+8(-1) \quad \boxed{\because i^2=-1}$$
$$= 15+22i-8$$
$$= 7+22i$$
$$|z_1 z_2| = |7+22i|$$
$$= \sqrt{(7)^2+(22)^2}$$
$$= \sqrt{49+484}$$
$$|z_1 z_2| = \sqrt{533}$$

7. Find real and imaginary parts of $z=(2+7i)^{-1}$. **10301157**

Solution:

Given that:

$z=(2+7i)^{-1}$

$z=\dfrac{1}{2+7i}$

$z=\dfrac{1}{(2+7i)}\times\dfrac{(2-7i)}{(2-7i)}$

$=\dfrac{2-7i}{(2)^2-(7i)^2}$ $\boxed{\because (a+b)(a-b)=a^2-b^2}$

$=\dfrac{2-7i}{4-49i^2}$

$=\dfrac{2-7i}{4-49(-1)}$ $\boxed{\because i^2=-1}$

$=\dfrac{2-7i}{4+49}$

$z=\dfrac{2-7i}{53}$

$z=\dfrac{2}{53}-\dfrac{7}{53}i$

$\text{Re}\,(z)=\dfrac{2}{53},\ \text{Img}\,(z)=\dfrac{-7}{53}$

8. Solve the given simultaneous linear equations with complex coefficients for z and w: **10301158**

$iz+(2-i)w=4+i$

$iz+(3+i)w=3+3i$

Solution:

$iz+(2-i)w=4+i$ ………(i)

$iz+(3+i)w=3+3i$ ………(ii)

Subtracting equation (i) and (ii), we have

$$\begin{array}{r} iz+(2-i)w=\quad 4+i \\ \pm iz \pm(3+i)w=\pm 3\pm 3i \\ \hline (2-i)w-(3+i)w=1-2i \end{array}$$

$(2-i-3-i)w=1-2i$

$(-1-2i)w=1-2i$

$w=\dfrac{1-2i}{-1-2i}$

$w=\dfrac{1-2i}{-1(1+2i)}$

$w=\dfrac{-1+2i}{1+2i}$

$w=\dfrac{(-1+2i)}{(1+2i)}\times\dfrac{(1-2i)}{(1-2i)}$

$w=\dfrac{-1(1-2i)+2i(1-2i)}{(1)^2-(2i)^2}$

$w=\dfrac{-1+2i+2i-4i^2}{1-4i^2}$

$w=\dfrac{-1+4i-4(-1)}{1-4(-1)}$ $\boxed{\because i^2=-1}$

$w=\dfrac{-1+4i+4}{1+4}$

$w=\dfrac{3+4i}{5}$

$w=\dfrac{3}{5}+\dfrac{4}{5}i$

Put $w=\dfrac{3}{5}+\dfrac{4}{5}i$ in equation (i)

$iz+(2-i)w=4+i$

$iz+(2-i)\left(\dfrac{3}{5}+\dfrac{4}{5}i\right)=4+i$

$iz+(2-i)\left(\dfrac{3+4i}{5}\right)=4+i$

$iz+\dfrac{(2-i)(3+4i)}{5}=4+i$

$iz+\dfrac{2(3+4i)-i(3+4i)}{5}=4+i$

$iz+\dfrac{6+8i-3i-4i^2}{5}=4+i$

$iz+\dfrac{6+5i-4(-1)}{5}=4+i$ $\boxed{\because i^2=-1}$

$$iz + \frac{6+5i+4}{5} = 4+i$$
$$iz + \frac{10+5i}{5} = 4+i$$
$$iz + \frac{10}{5} + \frac{5}{5}i = 4+i$$
$$iz + 2 + i = 4+i$$
$$iz = 4-2+i-i$$
$$iz = 2$$
$$z = \frac{2}{i} \times \frac{-i}{-i}$$
$$z = \frac{-2i}{-i^2} = \frac{2i}{i^2}$$
$$z = \frac{2i}{-1}$$
$$z = -2i$$

Thus $\boxed{z = -2i,\ w = \frac{3}{5} + \frac{4}{5}i}$

9. Solve $(3-4i)(a+bi) = 1+0i$ and find the values of a and b. **10301159**

Solution:

$$(3-4i)(a+ib) = 1+0i$$
$$a+ib = \frac{1}{3-4i}$$
$$= \frac{1}{3-4i} \times \frac{3+4i}{3+4i}$$
$$= \frac{3+4i}{(3-4i)(3+4i)}$$
$$= \frac{3+4i}{(3)^2 - (4i)^2}$$

$\boxed{\because (a+b)(a-b) = a^2 - b^2}$

$$= \frac{3+4i}{9-16i^2}$$
$$= \frac{3+4i}{9-16(-1)} \qquad \boxed{\because i^2 = -1}$$
$$= \frac{3+4i}{9+16}$$
$$a+ib = \frac{3+4i}{25}$$
$$a+ib = \frac{3}{25} + \frac{4}{25}i$$

By comparing real and imaginary parts of both sides, we have

$$a = \frac{3}{25}, b = \frac{4}{25}$$

10. Solve the equation for x and y:

$$(3-2i)(x+yi) = 2(x-2yi) + 2i - 1$$

Solution: **10301160**

$$(3-2i)(x+yi) = 2(x-2yi) + 2i - 1$$
$$3(x+yi) - 2i(x+yi) = 2x - 4yi + 2i - 1$$
$$3x + 3yi - 2xi - 2yi^2 = 2x - 1 + 2i - 4yi$$
$$3x + 3yi - 2xi - 2y(-1) = (2x-1) + i(2-4y)$$

$\boxed{\because i^2 = -1}$

$$(3x+2y) + i(3y-2x) = (2x-1) + i(2-4y)$$

By comparing real and imaginary parts of both sides, we have

$$3x + 2y = 2x - 1$$
$$3x + 2y - 2x = -1$$
$$x + 2y = -1 \qquad \text{.........(i)}$$

Now, $3y - 2x = 2 - 4y$

$$3y - 2x + 4y = 2$$
$$7y - 2x = 2$$
$$-2x + 7y = 2 \qquad \text{.........(ii)}$$

Multiply equation (i) by 2 and adding in equation (ii)

$$2x + 4y = -2$$
$$-2x + 7y = 2$$
$$11y = 0$$
$$y = 0$$

Put $y = 0$ in equation (i)

$$x + 2(0) = -1$$
$$x + 0 = -1$$
$$x = -1$$

Thus $\boxed{x = -1, y = 0}$

ADDITIONAL MCQS

1. Which of the following equations give imaginary roots? 10301161

(a) $x^2-1=0$ (b) $x^2+1=0$
(c) $x-1=0$ (d) $x+1=0$

2. $\sqrt{-1}=...$ 10301162

(a) i (b) $-i^2$
(c) $-i$ (d) i^2

3. Evaluating i^7 we get: 10301163

(a) i (b) 1
(c) $-i$ (d) -1

4. $\sqrt{-2}=...$ 10301064

(a) $\sqrt{2}i$ (b) $-\sqrt{2}i$
(c) $2i$ (d) $-2i$

5. In $x+yi$, if $x=0$, then complex number is said to be…number. 10301165

(a) Real (b) Purely imaginary
(c) Irrational (d) Rational

6. If $\alpha+\beta i=-2-5i$ then β is: 10301166

(a) -2 (b) 5
(c) -5 (d) 2

Basic Operations on Complex Numbers

7. Adding $3+4i$ and $-2-5i$ we get:

(a) $5+2i$ (b) $-1-i$ 10301167
(c) $-1+i$ (d) $1-i$

8. For $z_1=4-6i$, $z_2=3+2i$ and $z_1\cdot z_2=2z$ then $z=$ 10301168

(a) $-5+8i$ (b) $12+5i$
(c) $12-5i$ (d) $-12-5i$

9. If $z_1-z_2=4+6i$ and $z_2=3-2i$ then z_1 is: 10301169

(a) $-7+4i$ (b) $7-4i$
(c) $-7-4i$ (d) $7+4i$

10. If $z_1=4i$ and $z_2=3-9i$ then $z_1+z_2=$

(a) $3-5i$ (b) $3i-5$ 10301170
(c) $7-9i$ (d) $3+5i$

11. $\sqrt{-5}\times\sqrt{-20}=...$ 10301171

(a) 10 (b) -10
(c) -25 (d) $\sqrt{100}$

Properties of Complex Numbers

12. The additive inverse of $7-10i$ is:

(a) $-7-10i$ (b) $7+10i$ 10301172
(c) $-7+10i$ (d) $-10+7i$

13. The multiplicative inverse of $4-3i$ is:

(a) $\frac{4}{25}+\frac{3}{25}i$ (b) $\frac{4}{25}-\frac{3}{25}i$ 10301173
(c) $\frac{3}{25}+\frac{4}{25}i$ (d) $\frac{3}{25}-\frac{4}{25}i$

14. Which of the following is the multiplicative identity of complex numbers? 10301174

(a) $0+0i$ (b) $1+0i$
(c) $0+1i$ (d) $1+1i$

15. Which of the following property does not exist in complex numbers? 10301175

(a) Additive inverse
(b) Multiplicative inverse
(c) Multiplicative identity
(d) Order property

Complex Conjugates and Its Properties

16. Conjugate of $-5-7i$ is: 10301176

(a) $-5+7i$ (b) $5-7i$
(c) $5+7i$ (d) $-5-7i$

17. If $z=3-5i$ then $\bar{z}$ is: 10301177

(a) $-3-5i$ (b) $3-5i$
(c) $-3-5i$ (d) $3+5i$

18. The sum of a complex number and its conjugate is a/an ____ number. 10301178

(a) Real (b) Irrational
(c) Imaginary (d) Zero

19. The difference of a complex number from its conjugate is a/an ____ number.

(a) Real (b) Irrational 10301179
(c) Imaginary (d) Zero

20. The product of a complex number with its conjugate is a/an ____ number.

(a) Real (b) Irrational 10301180
(c) Imaginary (d) Zero

Modulus of a Complex Number

21. Modulus of complex number is the distance of a point from: **10301181**

(a) $x-axis$ (b) $y-axis$
(c) Origin (d) Infinity

22. Modulus of complex number $z=x+yi$ is: **10301182**

(a) $|z|=\sqrt{x^2+y^2}$ (b) $|z|=\sqrt{x^2-y^2}$
(c) $|z|=-\sqrt{x^2+y^2}$ (d) $|z|=\sqrt{-x^2-y^2}$

23. Modulus of $-3i+4$ is: **10301183**

(a) $1i$ (b) -5
(c) 25 (d) 5

24. If z_1 and z_2 are complex numbers, then $\overline{z_1\cdot z_2}=...$ **10301184**

(a) $\overline{z_1}\cdot\overline{z_2}$ (b) $\overline{z_1}\cdot z_2$
(c) $z_1\cdot\overline{z_2}$ (d) $\overline{\overline{z_1\cdot z_2}}$

25. $z\cdot\overline{z}=$ _________. **10301185**

(a) 0 (b) $|z|^2$
(c) 1 (d) z

Real and Imaginary Parts of Complex Number

26. Real part of $2ab(i+i^2)$ is: **10301186**

(a) $2ab$ (b) $-2ab$
(c) $2abi$ (d) $-2abi$

27. Imaginary part of $-i(3i+2)$ is: **10301187**

(a) -2 (b) 2
(c) 3 (d) -3

28. Real part of $(x+yi)^{-1}$ is: **10301188**

(a) $\frac{x}{x+y}$ (b) $\frac{x}{x^2+y^2}$
(c) $\frac{-x}{x^2+y^2}$ (d) $\frac{x}{x^2-y^2}$

29. Imaginary part of $(x+yi)^{-1}$ is: **10301189**

(a) $\frac{y}{x+y}$ (b) $\frac{y}{x^2+y^2}$
(c) $\frac{-y}{x^2+y^2}$ (d) $\frac{y}{x^2-y^2}$

30. Imaginary part of $(3-4i)^{-1}$ is: **10301190**

(a) $\frac{3}{25}$ (b) $\frac{4}{25}$
(c) $\frac{-4}{25}$ (d) $\frac{-3}{25}$

ANSWER KEY

1.	b	2.	a	3.	c	4.	a	5.	b	6.	c
7.	d	8.	c	9.	d	10.	a	11.	b	12.	c
13.	a	14.	b	15.	d	16.	a	17.	d	18.	a
19.	c	20.	a	21.	c	22.	a	23.	d	24.	a
25.	b	26.	b	27.	a	28.	b	29.	c	30.	b