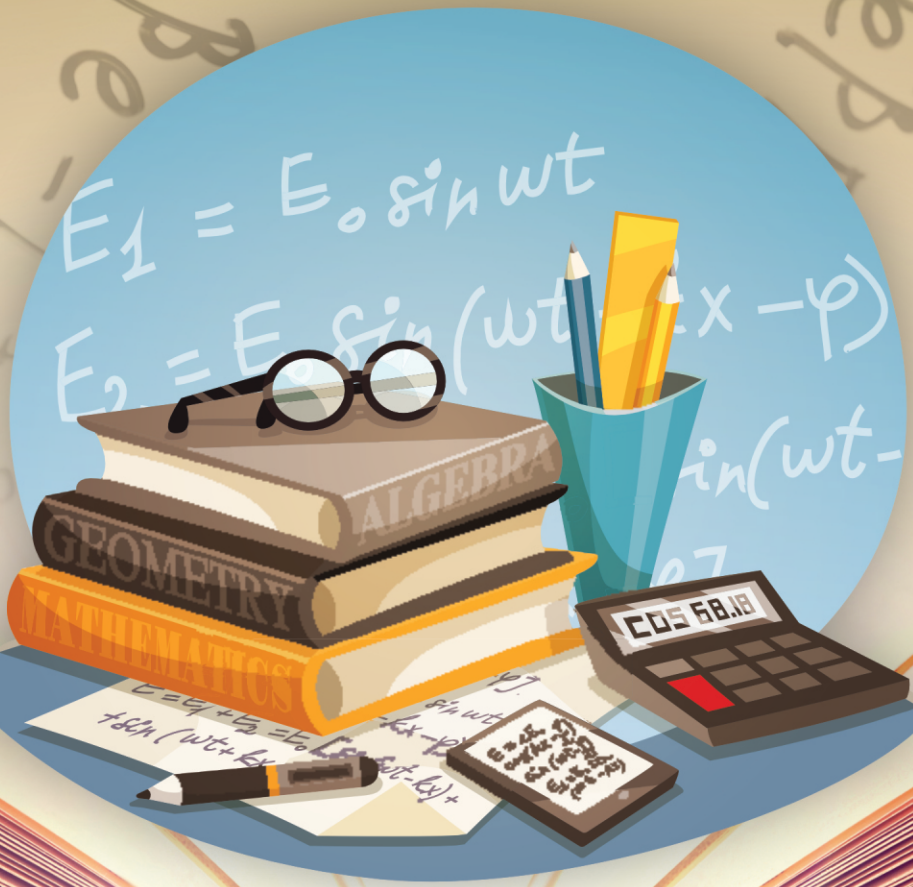
1st Choice of Position Holders

UNIQUE NOTES

MATHEMATICS 10

According to New Board Pattern



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CONTENTS

10th Mathematics

UNIT 01 Quadratic Equations

Q. Define Quadratic Equation and write its standard form. **(Board 2014, 15, 18)** 10301001

Ans. **Quadratic Equation**

An Equation, which contains the square of the unknown (variable) quantity, but not higher power, is called a quadratic equation or an equation of the second degree.

Standard form:

A second degree equation in one variable x of the form $ax^2+bx+c=0$, where $a \neq 0$ and a, b, c are real numbers is called the general or standard form of a quadratic equation.

Q. Define Pure Quadratic Equation. 10301002

Ans. **Pure Quadratic Equation** **(Board 2019)**

If b = 0 in quadratic equation $ax^2+bx+c=0$ then it is called a pure quadratic equation.

i.e $ax^2+0x+c=0$

$ax^2+c=0$

e.g $x^2-16=0$

Q. Write the name of methods to solve the Quadratic Equations. **(Board 2016, 17)** 10301003

Ans. **Solution of Quadratic Equations.**

To find solution set of a quadratic equation following methods are used:

(i) Factorization
(ii) Completing Square
(iii) Quadratic formula

Example 1:

Solve the quadratic equation $3x^2-6x = x + 20$ by Factorization. 10301004

Solution: $3x^2-6x = x + 20$

$3x^2 - 6x - x - 20 = 0$

$3x^2 - 7x - 20 = 0$

$3x^2 - 12x + 5x - 20 = 0$

$3x(x - 4) + 5(x - 4) = 0$

$(x - 4)(3x + 5) = 0$

Either $x - 4 = 0$ or $3x + 5 = 0$

$x = 4$ or $3x = -5$

or $x=\frac{-5}{3}$

Thus, the solution set is $\left\{\frac{-5}{3},4\right\}$

Example 2:

Solve $5x^2 = 30x$ by factorization 10301005

Solution: $5x^2 = 30x$ **(Board 2019)**

$5x^2 - 30x = 0$

Which is factorized as

$5x(x - 6) = 0$

Either $5x = 0$ or $x - 6 = 0$

$x=\frac{0}{5}$ or $x = 6$

$x = 0$ or $x = 6$

Thus the solution set is {0, 6}

Solution by Completing Square

Example 1: Solve the equation $x^2-3x-4 = 0$ by completing square. **(Board 2024)** 10301006

Solution: $x^2 - 3x - 4 = 0$

Shifting constant term –4 to the right, we have

$x^2 - 3x = 4$

$(x)^2 - (x)(3) = 4$

$$x^2-2(x)\left(\frac{3}{2}\right)=4$$

Adding $\left(\frac{3}{2}\right)^2$ on the both sides we get

$$(x)^2-2(x)\left(\frac{3}{2}\right)+\left(\frac{3}{2}\right)^2=4+\left(\frac{3}{2}\right)^2$$

$$\left(x-\frac{3}{2}\right)^2=4+\frac{9}{4}$$

$$\left(x-\frac{3}{2}\right)^2=\frac{16+9}{4}$$

$$\left(x-\frac{3}{2}\right)^2=\frac{25}{4}$$

Taking square root of both sides, we get

$$\sqrt{\left(x-\frac{3}{2}\right)^2}=\pm\sqrt{\frac{25}{4}}$$

$$\left(x-\frac{3}{2}\right)=\pm\frac{5}{2}$$

$$x=\frac{3}{2}\pm\frac{5}{2}$$

$$x=\frac{3\pm5}{2}$$

$\Rightarrow \quad x=\frac{3+5}{2} \quad$ or $\quad x=\frac{3-5}{2}$

Either $x=\frac{8}{2} \quad$ or $\quad x=\frac{-2}{2}$

$\Rightarrow \quad x = 4 \quad$ or $\quad x = -1$

$\therefore$ 4 and –1 are the roots of the given equation. Thus the solution set is {–1, 4}

Example 2:

Solve the equation $2x^2-5x-3=0$ by completing square. 10301007

Solution: $2x^2 - 5x - 3 = 0$

Dividing each term by 2

$$x^2-\frac{5}{2}x-\frac{3}{2}=0$$

Shifting constant term $\frac{-3}{2}$ to the right

$$x^2-\frac{5}{2}x=\frac{3}{2}$$

$$x^2-(x)\left(\frac{5}{2}\right)=\frac{3}{2}$$

$$x^2-\frac{2(x)}{2}\left(\frac{5}{2}\right)=\frac{3}{2}$$

$$(x)^2-2(x)\left(\frac{5}{4}\right)=\frac{3}{2}$$

Adding $\left(\frac{5}{4}\right)^2$ on both sides we get

$$(x)^2-2(x)\left(\frac{5}{4}\right)+\left(\frac{5}{4}\right)^2=\frac{3}{2}+\left(\frac{5}{4}\right)^2$$

$$\left(x-\frac{5}{4}\right)^2=\frac{3}{2}+\frac{25}{16}$$

$$\left(x-\frac{5}{4}\right)^2=\frac{24+25}{16}$$

$$\left(x-\frac{5}{4}\right)^2=\frac{49}{16}$$

Taking square root on both sides, we get

$$\sqrt{\left(x-\frac{5}{4}\right)^2}=\pm\sqrt{\frac{49}{16}}$$

$$x-\frac{5}{4}=\pm\frac{7}{4}$$

$$x=\frac{5}{4}\pm\frac{7}{4}$$

$$x=\frac{5\pm7}{4}$$

$\Rightarrow \quad x=\frac{5+7}{4} \quad$ or $\quad x=\frac{5-7}{4}$

$x=\frac{12}{4} \quad$ or $\quad x=\frac{-2}{4}$

$x = 3 \quad$ or $\quad x=\frac{-1}{2}$

$\therefore \quad x=\frac{-1}{2}$, 3 are the roots of the given equation. Thus the solution set is $\left\{\frac{-1}{2}, 3\right\}$

EXERCISE 1.1

Q.1 Write the following quadratic equations in the Standard form and point out pure quadratic equations.

(i) $\mathbf{(x+7)(x-3)=-7}$ **(Board 2020,22)** 10301008

Solution: $\mathbf{(x+7)(x-3)=-7}$

$$x^2-3x+7x-21=-7$$

$$x^2+4x-21+7=0$$

$$x^2+4x-14=0$$

The standard form of quadratic equation is:

$$x^2+4x-14=0$$

(ii) $\mathbf{\frac{x^2+4}{3}-\frac{x}{7}=1}$ 10301009

Solution: $\frac{x^2+4}{3}-\frac{x}{7}=1$

$$\frac{7(x^2+4)-3x}{21}=1$$

$$7x^2+28-3x=21$$

$$7x^2-3x+28-21=0$$

$$7x^2-3x+7=0$$

The standard form of quadratic equation is:

$$7x^2-3x+7=0$$

(iii) $\dfrac{x}{x+1}+\dfrac{x+1}{x}=6$ **(Board 2017,18,24)** 10301010

Solution: $\dfrac{x}{x+1}+\dfrac{x+1}{x}=6$

$$\frac{x^2+(x+1)^2}{x(x+1)}=6$$
$$x^2+x^2+1+2x=6x(x+1)$$
$$2x^2+2x+1=6x^2+6x$$
$$0=6x^2+6x-2x^2-2x-1$$
$$0=4x^2+4x-1$$
$$\Rightarrow \quad 4x^2+4x-1=0$$

The standard form of quadratic equation is:
$$4x^2+4x-1=0$$

(iv) $\left(\dfrac{x+4}{x-2}\right)-\left(\dfrac{x-2}{x}\right)+4=0$ 10301011 **(Board 2022,23)**

Solution: $\left(\dfrac{x+4}{x-2}\right)-\left(\dfrac{x-2}{x}\right)+4=0$

$$\frac{(x+4)x-(x-2)^2+4x(x-2)}{(x-2)(x)}=0$$
$$x^2+4x-[x^2+2^2-2(x)(2)]+4x^2-8x=0(x-2)(x)$$
$$\cancel{x^2}+4x-\cancel{x^2}-4+4x+4x^2-8x=0$$
$$4x^2+4x+4x-8x-4=0$$
$$4x^2+\cancel{8x}-\cancel{8x}-4=0$$
$$4x^2-4=0$$
$$4(x^2-1)=0$$
$$\therefore \quad x^2-1=0 \quad (\because 4\neq 0)$$

So, $x^2-1=0$ is Pure Quadratic Equation

(v) $\dfrac{x+3}{x+4}-\dfrac{x-5}{x}=1$ 10301012

Solution: $\dfrac{x+3}{x+4}-\dfrac{x-5}{x}=1$

$$\frac{x(x+3)-(x+4)(x-5)}{x(x+4)}=1$$
$$x^2+3x-(x^2-5x+4x-20)=1x(x+4)$$
$$x^2+3x-(x^2-1x-20)=x^2+4x$$
$$\cancel{x^2}+3x-\cancel{x^2}+x+20=x^2+4x$$
$$3x+x+20=x^2+4x$$
$$4x+20=x^2+4x$$
$$\Rightarrow \quad x^2+\cancel{4x}-\cancel{4x}-20=0$$
$$x^2-20=0$$
$$x^2+0x-20=0$$
$$\text{As, } b=0$$

So, $x^2-20=0$ is Pure Quadratic Equation

(vi) $\dfrac{x+1}{x+2}+\dfrac{x+2}{x+3}=\dfrac{25}{12}$ 10301013

Solution:

$$\frac{x+1}{x+2}+\frac{x+2}{x+3}=\frac{25}{12}$$
$$\frac{(x+1)(x+3)+(x+2)(x+2)}{(x+2)(x+3)}=\frac{25}{12}$$
$$\frac{(x^2+3x+1x+3)+(x^2+2x+2x+4)}{(x+2)(x+3)}=\frac{25}{12}$$
$$\frac{(x^2+4x+3)+(x^2+4x+4)}{x^2+3x+2x+6}=\frac{25}{12}$$
$$\frac{2x^2+8x+7}{x^2+5x+6}=\frac{25}{12}$$
$$12(2x^2+8x+7)=25(x^2+5x+6)$$
$$24x^2+96x+84=25x^2+125x+150$$
$$25x^2-24x^2+125x-96x+150-84=0$$
$$x^2+29x+66=0$$

The standard form of quadratic equation is:
$$x^2+29x+66=0$$

Q.2 Solve by Factorization: 10301014

(i) $x^2-x-20=0$ **(Board 2014,15,16,18,24)**

Solution:

$$x^2-x-20=0$$
$$x^2-5x+4x-20=0$$
$$x(x-5)+4(x-5)=0$$
$$(x-5)(x+4)=0$$
$$x-5=0 \quad \text{or} \quad x+4=0$$
$$x=5 \quad \text{or} \quad x=-4$$

Solution set is $\{-4, 5\}$

(ii) $3y^2=y(y-5)$ **(Board 2024)** 10301015

Solution:

$$3y^2=y(y-5)$$
$$3y^2=y^2-5y$$
$$3y^2-y^2+5y=0$$
$$2y^2+5y=0$$
$$y(2y+5)=0$$

Either $y=0$ or $2y+5=0$

$y=0$ or $2y=-5$

$y=0$ or $y=\dfrac{-5}{2}$

Solution is $\left\{0, \dfrac{-5}{2}\right\}$

(iii) $\mathbf{4-32x=17x^2}$ 10301016

Solution: $4-32x=17x^2$

$17x^2+32x-4=0$

$17x^2+34x-2x-4=0$

$17x(x+2)-2(x+2)=0$

$(x+2)(17x-2)=0$

Either $x+2=0$ or $17x-2=0$

$x=-2$ or $17x=2$

$x=-2$ or $x=\frac{2}{17}$

Solution set is $\left\{-2, \frac{2}{17}\right\}$

(iv) $\mathbf{x^2-11x=152}$ **(Board 2021,23)** 10301017

Solution: $x^2-11x=152$

$x^2-11x-152=0$

$x^2-19x+8x-152=0$

$x(x-19)+8(x-19)=0$

$(x-19)(x+8)=0$

$x-19=0$ or $x+8=0$

$x=19$ or $x=-8$

Solution set is $\{-8, 19\}$

(v) $\mathbf{\frac{x+1}{x}+\frac{x}{x+1}=\frac{25}{12}}$ 10301018 **(Board 2021)**

Solution: $\frac{x+1}{x}+\frac{x}{x+1}=\frac{25}{12}$

$\frac{(x+1)^2+x^2}{x(x+1)}=\frac{25}{12}$

$\frac{(x)^2+(1)^2+2(x)(1)+x^2}{x^2+x}=\frac{25}{12}$

$\frac{x^2+1+2x+x^2}{x^2+x}=\frac{25}{12}$

$12(2x^2+2x+1)=25(x^2+x)$

$24x^2+24x+12=25x^2+25x$

$0=25x^2+25x-24x^2-24x-12$

$0=25x^2-24x^2+25x-24x-12$

$0=x^2+x-12$

$\Rightarrow x^2+x-12=0$

$x^2+4x-3x-12=0$

$x(x+4)-3(x+4)=0$

$(x+4)(x-3)=0$

Either

$x+4=0$ or $x-3=0$

$\Rightarrow x=-4$ or $x=3$

Solution set is $\{3, -4\}$

(vi) $\mathbf{\frac{2}{x-9}=\frac{1}{x-3}-\frac{1}{x-4}}$ 10301019

Solution: $\frac{2}{x-9}=\frac{1}{x-3}-\frac{1}{x-4}$

$\frac{2}{x-9}=\frac{(x-4)-(x-3)}{(x-3)(x-4)}$

$\frac{2}{x-9}=\frac{\cancel{x}-4-\cancel{x}+3}{x^2-4x-3x+12}$

$\frac{2}{x-9}=\frac{-1}{x^2-7x+12}$

$2(x^2-7x+12)=-1(x-9)$

$2x^2-14x+24=-1x+9$

$2x^2-14x+x+24-9=0$

$2x^2-13x+15=0$

$2x^2-10x-3x+15=0$

$2x(x-5)-3(x-5)=0$

$(x-5)(2x-3)=0$

Either $x-5=0$ or $2x-3=0$

$x=5$ or $2x=3$

$x=5$ or $x=\frac{3}{2}$

Solution set is $\left\{5, \frac{3}{2}\right\}$

Q.3 Solve the following equations by completing square. 10301020

(i) $\mathbf{7x^2+2x-1=0}$ **(Board 2015,19,21)**

Solution: $7x^2+2x-1=0$

Dividing each term of the equation by 7

$\frac{\cancel{7}}{\cancel{7}}x^2+\frac{2}{7}x-\frac{1}{7}=\frac{0}{7}$

$x^2+\frac{2}{\cancel{2}}(x)\left(\frac{\cancel{2}}{7}\right)^1-\frac{1}{7}=0$

$(x)^2+2(x)\left(\frac{1}{7}\right)=\frac{1}{7}$

Adding $\left(\frac{1}{7}\right)^2$ on both sides

$(x)^2+2(x)\left(\frac{1}{7}\right)+\left(\frac{1}{7}\right)^2=\frac{1}{7}+\left(\frac{1}{7}\right)^2$

$\left(x+\frac{1}{7}\right)^2=\frac{1}{7}+\frac{1}{49}$

$$\left(x+\frac{1}{7}\right)^2=\frac{7+1}{49}$$

$$\left(x+\frac{1}{7}\right)^2=\frac{8}{49}$$

Taking square root of both sides

$$\sqrt{\left(x+\frac{1}{7}\right)^2}=\pm\sqrt{\frac{8}{49}}$$

$$\sqrt{\left(x+\frac{1}{7}\right)^2}=\pm\sqrt{\frac{4\times2}{49}}$$

$$x+\frac{1}{7}=\pm\frac{2\sqrt{2}}{7}$$

$$x=\frac{-1}{7}\pm\frac{2\sqrt{2}}{7}$$

$$\therefore \quad x=\frac{-1\pm2\sqrt{2}}{7}$$

Solution set is $\left\{\frac{-1\pm2\sqrt{2}}{7}\right\}$

(ii) $\mathbf{ax^2+4x-a=0}$, $a\neq0$ 10301021 **(Board 2022)**

Solution: $ax^2+4x-a=0$, $a\neq0$

Dividing each term of the equation by 'a'

$$\frac{ax^2}{a}+4\frac{x}{a}-\frac{a}{a}=\frac{0}{a}$$

$$x^2+4\frac{x}{a}-1=0$$

$$x^2+\frac{2}{\cancel{2}}(x)\left(\frac{\cancel{4}^{2}}{a}\right)=1$$

$$(x)^2+2(x)\left(\frac{2}{a}\right)=1$$

Adding $\left(\frac{2}{a}\right)^2$ on both sides

$$(x)^2+2(x)\left(\frac{2}{a}\right)+\left(\frac{2}{a}\right)^2=1+\left(\frac{2}{a}\right)^2$$

$$\left(x+\frac{2}{a}\right)^2=1+\frac{4}{a^2}$$

$$\left(x+\frac{2}{a}\right)^2=\frac{a^2+4}{a^2}$$

Taking square root of both sides

$$\sqrt{\left(x+\frac{2}{a}\right)^2}=\pm\sqrt{\frac{a^2+4}{a^2}}$$

$$x+\frac{2}{a}=\pm\frac{\sqrt{a^2+4}}{a}$$

$$x=\frac{-2}{a}\pm\frac{\sqrt{a^2+4}}{a}$$

$$x=\frac{-2\pm\sqrt{a^2+4}}{a}$$

Solution set is $\left\{\frac{-2\pm\sqrt{a^2+4}}{a}\right\}$

(iii) $\mathbf{11x^2-34x+3=0}$ **(Board 2017)** 10301022

Solution: $11x^2-34x+3=0$

Dividing each term of the equation by 11

$$\frac{\cancel{11}}{\cancel{11}}x^2-\frac{34}{11}x+\frac{3}{11}=0$$

$$x^2-\frac{34}{11}x=-\frac{3}{11}$$

$$(x)^2-\frac{2}{\cancel{2}}(x)\left(\frac{\cancel{34}^{17}}{11}\right)=-\frac{3}{11}$$

$$(x)^2-2(x)\left(\frac{17}{11}\right)=-\frac{3}{11}$$

Adding $\left(\frac{17}{11}\right)^2$ on both sides we get

$$(x)^2-2(x)\left(\frac{17}{11}\right)+\left(\frac{17}{11}\right)^2=-\frac{3}{11}+\left(\frac{17}{11}\right)^2$$

$$\left(x-\frac{17}{11}\right)^2=\frac{-3}{11}+\frac{289}{121}$$

$$\left(x-\frac{17}{11}\right)^2=\frac{-33+289}{121}$$

$$\left(x-\frac{17}{11}\right)^2=\frac{256}{121}$$

Taking square root on both sides

$$\sqrt{\left(x-\frac{17}{11}\right)^2}=\pm\sqrt{\frac{256}{121}}$$

$$x-\frac{17}{11}=\pm\frac{16}{11}$$

$$x=\frac{17}{11}\pm\frac{16}{11}$$

$$x = \frac{17 \pm 16}{11}$$

$$\Rightarrow \quad x = \frac{17-16}{11} \quad \text{or} \quad x = \frac{17+16}{11}$$

$$x = \frac{1}{11} \quad \text{or} \quad x = \frac{33}{11}$$

$$x = 3$$

Solution set is $\left\{3, \frac{1}{11}\right\}$

(iv) $\ell x^2 + mx + n = 0$, $\ell \neq 0$ 10301023

Solution: $\ell x^2 + mx + n = 0$, $\ell \neq 0$

Dividing each term of the equation by ℓ

$$\frac{\cancel{\ell} x^2}{\cancel{\ell}} + (x)\left(\frac{m}{\ell}\right)x = -\frac{n}{\ell}$$

$$x^2 + \frac{2}{2}(x)\left(\frac{m}{\ell}\right) = -\frac{n}{\ell}$$

$$x^2 + 2(x)\left(\frac{m}{2\ell}\right) = -\frac{n}{\ell}$$

Adding $\left(\frac{m}{2\ell}\right)^2$ on both sides

$$(x)^2 + 2(x)\left(\frac{m}{2\ell}\right) + \left(\frac{m}{2\ell}\right)^2 = -\frac{n}{\ell} + \left(\frac{m}{2\ell}\right)^2$$

$$\left(x + \frac{m}{2\ell}\right)^2 = -\frac{n}{\ell} + \frac{m^2}{4\ell^2}$$

$$\left(x + \frac{m}{2\ell}\right)^2 = \frac{-4\ell n + m^2}{4\ell^2}$$

Taking square root on both sides

$$\sqrt{\left(x + \frac{m}{2\ell}\right)^2} = \pm\sqrt{\frac{m^2 - 4\ell n}{4\ell^2}}$$

$$x + \frac{m}{2\ell} = \pm\frac{\sqrt{m^2 - 4\ell n}}{2\ell}$$

$$x = -\frac{m}{2\ell} \pm \frac{\sqrt{m^2 - 4\ell n}}{2\ell}$$

$$x = \frac{-m \pm \sqrt{m^2 - 4\ell n}}{2\ell}$$

Solution set is $\left\{\frac{-m \pm \sqrt{m^2 - 4\ell n}}{2\ell}\right\}$

(v) $3x^2 + 7x = 0$ **(Board 2014)** 10301024

Solution: $3x^2 + 7x = 0$

Dividing each term of equation by 3

$$\frac{\cancel{3}x^2}{\cancel{3}} + \frac{7}{3}x = 0$$

$$x^2 + \frac{2}{2}(x)\left(\frac{7}{3}\right) = 0$$

$$(x)^2 + 2(x)\left(\frac{7}{6}\right) = 0$$

Adding $\left(\frac{7}{6}\right)^2$ on both sides

$$(x)^2 + 2(x)\left(\frac{7}{6}\right) + \left(\frac{7}{6}\right)^2 = \left(\frac{7}{6}\right)^2$$

$$\left(x + \frac{7}{6}\right)^2 = \frac{49}{36}$$

Taking square root on both sides

$$\sqrt{\left(x + \frac{7}{6}\right)^2} = \pm\sqrt{\frac{49}{36}}$$

$$\left(x + \frac{7}{6}\right) = \pm\frac{7}{6}$$

$$x = \frac{-7}{6} \pm \frac{7}{6}$$

$$x = \frac{-7 \pm 7}{6}$$

$$\Rightarrow \quad x = \frac{-7-7}{6} \quad \text{or} \quad x = \frac{-\cancel{7}+\cancel{7}}{6}$$

$$x = \frac{-14}{6} \quad \text{or} \quad x = \frac{0}{6}$$

$$x = \frac{-7}{3} \quad \text{or} \quad x = 0$$

Solution set is $\left\{0, \frac{-7}{3}\right\}$

(vi) $x^2 - 2x - 195 = 0$ **(Board 2014,20)** 10301025

Solution: $x^2 - 2x - 195 = 0$

$$x^2 - 2x = 195$$

$$x^2 - \frac{2}{\cancel{2}}(x)\left(\overset{1}{\cancel{2}}\right) = 195$$

$$(x)^2 - 2(x)(1) = 195$$

Adding $(1)^2$ on both sides

$$(x)^2 - 2(x)(1) + (1)^2 = 195 + (1)^2$$

$$(x-1)^2 = 195+1$$
$$(x-1)^2 = 196$$

Taking square root on both sides

$$\sqrt{(x-1)^2} = \pm\sqrt{196}$$
$$x-1 = \pm 14$$
$$x = 1 \pm 14$$
$$\Rightarrow \quad x = 1-14 \quad \text{or} \quad x = 1+14$$
$$x = -13 \quad \text{or} \quad x = 15$$

Solution set is $\{-13, 15\}$

(vii) $\mathbf{-x^2 + \frac{15}{2} = \frac{7}{2}x}$ 10301026

Solution: $-x^2 + \frac{15}{2} = \frac{7}{2}x$

$$\frac{15}{2} = x^2 + \frac{7}{2}x$$
$$\Rightarrow \quad x^2 + \frac{7}{2}x = \frac{15}{2}$$
$$x^2 + \frac{2}{2}(x)\left(\frac{7}{2}\right) = \frac{15}{2}$$
$$(x)^2 + 2(x)\left(\frac{7}{4}\right) = \frac{15}{2}$$

Adding $\left(\frac{7}{4}\right)^2$ on both sides

$$(x)^2 + 2(x)\left(\frac{7}{4}\right) + \left(\frac{7}{4}\right)^2 = \frac{15}{2} + \left(\frac{7}{4}\right)^2$$
$$\left(x+\frac{7}{4}\right)^2 = \frac{15}{2} + \frac{49}{16}$$
$$\left(x+\frac{7}{4}\right)^2 = \frac{120+49}{16}$$
$$\left(x+\frac{7}{4}\right)^2 = \frac{169}{16}$$

Taking square root on both sides

$$\sqrt{\left(x+\frac{7}{4}\right)^2} = \pm\sqrt{\frac{169}{16}}$$
$$x + \frac{7}{4} = \pm\frac{13}{4}$$
$$x = \frac{-7}{4} \pm \frac{13}{4}$$
$$x = \frac{-7 \pm 13}{4}$$
$$\Rightarrow \quad x = \frac{-7-13}{4} \quad \text{or} \quad x = \frac{-7+13}{4}$$
$$x = \frac{-20}{4} \quad \text{or} \quad x = \frac{6}{4}$$
$$x = -5 \quad \text{or} \quad x = \frac{3}{2}$$

Solution set is $\left\{-5, \frac{3}{2}\right\}$

(viii) $\mathbf{x^2 + 17x + \frac{33}{4} = 0}$ 10301027

Solution: $x^2 + 17x + \frac{33}{4} = 0$

$$x^2 + 17x = -\frac{33}{4}$$
$$(x)^2 + 2(x)\left(\frac{17}{2}\right) = -\frac{33}{4}$$

Adding $\left(\frac{17}{2}\right)^2$ on both sides

$$(x)^2 + 2(x)\left(\frac{17}{2}\right) + \left(\frac{17}{2}\right)^2 = \frac{-33}{4} + \left(\frac{17}{2}\right)^2$$
$$\left(x+\frac{17}{2}\right)^2 = \frac{-33}{4} + \frac{289}{4}$$
$$\left(x+\frac{17}{2}\right)^2 = \frac{-33+289}{4}$$
$$\left(x+\frac{17}{2}\right)^2 = \frac{256}{4}$$

Taking square root

$$\sqrt{\left(x+\frac{17}{2}\right)^2} = \pm\sqrt{\frac{256}{4}}$$
$$x + \frac{17}{2} = \pm\frac{16}{2}$$
$$x = \frac{-17}{2} \pm \frac{16}{2}$$
$$x = \frac{-17 \pm 16}{2}$$
$$\Rightarrow \quad x = \frac{-17-16}{2} \quad \text{or} \quad x = \frac{-17+16}{2}$$
$$x = \frac{-33}{2} \quad \text{or} \quad x = \frac{-1}{2}$$

Solution set is $\left\{-\frac{1}{2}, \frac{-33}{2}\right\}$

(ix) $4-\dfrac{8}{3x+1}=\dfrac{3x^2+5}{3x+1}$ 10301028

Solution: $4-\dfrac{8}{3x+1}=\dfrac{3x^2+5}{3x+1}$

$$4=\frac{3x^2+5}{3x+1}+\frac{8}{3x+1}$$
$$4=\frac{3x^2+5+8}{3x+1}$$
$$4(3x+1)=3x^2+13$$
$$12x+4=3x^2+13$$
$$3x^2+13-12x-4=0$$
$$3x^2-12x+9=0$$

Dividing each term of the equation by 3

$$\frac{\cancel{3}}{\cancel{3}}x^2-\frac{\cancel{12}^{4}x}{\cancel{3}}+\frac{\cancel{9}^{3}}{\cancel{3}}=0$$
$$x^2-4x+3=0$$
$$x^2-4x=-3$$
$$(x)^2-2(x)(2)=-3$$

Adding $(2)^2$ on both sides

$$(x)^2-2(x)(2)+(2)^2=-3+(2)^2$$
$$(x-2)^2=-3+4$$
$$(x-2)^2=1$$

Taking square root

$$\sqrt{(x-2)^2}=\pm\sqrt{1}$$
$$x-2=\pm1$$
$$\Rightarrow\quad x-2=+1 \quad\text{or}\quad x-2=-1$$
$$x=2+1 \qquad\qquad x=2-1$$
$$x=3 \quad\text{or}\quad x=1$$

Solution set is $\{1,\ 3\}$

(x) $7(x+2a)^2+3a^2=5a(7x+23a)$ 10301029

Solution: $7(x+2a)^2+3a^2=5a(7x+23a)$

$$7(x^2+4a^2+4ax)+3a^2=35ax+115a^2$$
$$7x^2+28a^2+28ax+3a^2-35ax-115a^2=0$$
$$7x^2-35ax+28ax+28a^2+3a^2-115a^2=0$$
$$7x^2-7ax-84a^2=0$$

Dividing each term of the equation by 7 we get

$$\frac{\cancel{7}x^2}{\cancel{7}}-\frac{\cancel{7}ax}{\cancel{7}}=\frac{\cancel{84}^{12}a^2}{\cancel{7}}$$
$$x^2-ax=12a^2$$
$$(x)^2-2(x)\left(\frac{a}{2}\right)=12a^2$$

Adding $\left(\dfrac{a}{2}\right)^2$ on both sides

$$(x)^2-2(x)\left(\frac{a}{2}\right)+\left(\frac{a}{2}\right)^2=12a^2+\left(\frac{a}{2}\right)^2$$
$$\left(x-\frac{a}{2}\right)^2=12a^2+\frac{a^2}{4}$$
$$\left(x-\frac{a}{2}\right)^2=\frac{48a^2+a^2}{4}$$
$$\left(x-\frac{a}{2}\right)^2=\frac{49a^2}{4}$$

Taking square root on both sides

$$\sqrt{\left(x-\frac{a}{2}\right)^2}=\pm\sqrt{\frac{49a^2}{4}}$$
$$x-\frac{a}{2}=\pm\frac{7a}{2}$$
$$x=\frac{a}{2}\pm\frac{7a}{2}$$
$$x=\frac{a\pm7a}{2}$$
$$\Rightarrow\quad x=\frac{a+7a}{2} \quad\text{or}\quad x=\frac{a-7a}{2}$$
$$x=\frac{8a}{2} \quad\text{or}\quad x=\frac{-6a}{2}$$
$$x=4a \quad\text{or}\quad x=-3a$$

Solution set is $\{-3a,\ 4a\}$

Q. Derive Quadratic Formula OR
Find the solution of $ax^2+bx+c=0,\ a\neq0$ by completing square method 10301030

<u>Derivation of Quadratic Formula:</u>

The quadratic equation in standard form is

$$ax^2+bx+c=0,\quad a\neq0$$

Dividing each term of the equation by 'a' we get.

$$\frac{\cancel{a}x^2}{\cancel{a}}+\frac{b}{a}x+\frac{c}{a}=0$$

Shifting constant term $\dfrac{c}{a}$ to the right, we have

$$x^2+\frac{b}{a}x=\frac{-c}{a}$$

$$x^2+\frac{2}{2}(x)\left(\frac{b}{a}\right)=-\frac{c}{a}$$

$$(x)^2+2(x)\left(\frac{b}{2a}\right)=\frac{-c}{a}$$

Adding $\left(\frac{b}{2a}\right)^2$ on both sides we obtain

$$(x)^2+2(x)\left(\frac{b}{2a}\right)+\left(\frac{b}{2a}\right)^2=\left(\frac{b}{2a}\right)^2-\frac{c}{a}$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a}$$

or $$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

Taking square root of the both sides, we get

$$\sqrt{\left(x+\frac{b}{2a}\right)^2}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$\Rightarrow \quad x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow \quad x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

or $$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Thus, $\boldsymbol{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$, $a \neq 0$ is known as "quadratic Formula"

Example 1: Solve the Quadratic equation $2 + 9x = 5x^2$ by using quadratic formula.

Solution: $2 + 9x = 5x^2$ 10301031

The given equation in standard form can be written as

$$5x^2 - 9x - 2 = 0$$

Comparing with $ax^2 + bx + c = 0$, we get

$$a = 5, \quad b = -9, \quad c = -2$$

Putting the values in quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}, \quad \text{we have}$$

$$x=\frac{-(-9)\pm\sqrt{(-9)^2-4(5)(-2)}}{2(5)}$$

Or $$x=\frac{9\pm\sqrt{81+40}}{10}=\frac{9\pm\sqrt{121}}{10}=\frac{9\pm 11}{10}$$

Either $x=\frac{9+11}{10}$ or $x=\frac{9-11}{10}$

$x=\frac{20}{10}$ or $x=\frac{-2}{10}$

$x=2$ or $x=\frac{-1}{5}$

Thus, the solution set is $\left\{\frac{-1}{5}, 2\right\}$

Example 2: Using quadratic formula, solve the equation $\frac{2x+1}{x+2}-\frac{x-2}{x+4}=0$ 10301032

Solution: $\frac{2x+1}{x+2}-\frac{x-2}{x+4}=0$

Simplifying and writing in the standard form

$$\frac{(2x+1)(x+4)-(x-2)(x+2)}{(x+2)(x+4)}=0$$

$$(2x + 1)(x + 4) - [(x^2 - 2^2)] = 0$$

$$2x^2 + 8x + x + 4 - (x^2 - 4) = 0$$

$$2x^2 + 9x + 4 - x^2 + 4 = 0$$

Or $x^2 + 9x + 8 = 0$

$$ax^2+bx+c=0$$

Here $a = 1, \quad b = 9, \quad c = 8$

Using quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}, \text{ we have}$$

$$x=\frac{-9\pm\sqrt{(9)^2-4\times1\times8}}{2\times1}$$

$$x=\frac{-9\pm\sqrt{81-32}}{2}$$

$$x=\frac{-9\pm\sqrt{49}}{2}=\frac{-9\pm7}{2}$$

$x=\frac{-9+7}{2}$ | $x=\frac{-9-7}{2}$

$x=\frac{-2}{2}=-1$ | $x=\frac{-16}{2}=-8$

Thus, the solution set is $\{-8, -1\}$

EXERCISE 1.2

Q.1 Solve the following equations using quadratic formula:

(i) $2 - x^2 = 7x$ **(Board 2020,22)** 10301033

Solution: $2 - x^2 = 7x$

$1x^2 + 7x - 2 = 0$

As $ax^2 + bx + c = 0$

$\Rightarrow$ $a = 1,\ b = 7,\ c = -2$

Using Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(7) \pm \sqrt{(7)^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{-7 \pm \sqrt{49+8}}{2} \Rightarrow x = \frac{-7 \pm \sqrt{57}}{2}$$

Solution set is $\left\{\frac{-7 \pm \sqrt{57}}{2}\right\}$

(ii) $5x^2 + 8x + 1 = 0$ 10301034

Solution: $5x^2 + 8x + 1 = 0$

As $ax^2 + bx + c = 0$

$\Rightarrow$ $a = 5$, $b = 8$, $c = 1$

Using Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(5)(1)}}{2(5)}$$

$$x = \frac{-8 \pm \sqrt{64 - 20}}{10}$$

$$x = \frac{-8 \pm \sqrt{44}}{10}$$

$$x = \frac{-8 \pm \sqrt{4 \times 11}}{10}$$

$$x = \frac{-8 \pm 2\sqrt{11}}{10}$$

$$x = \frac{\cancel{2}\left(-4 \pm \sqrt{11}\right)}{\cancel{10}\ 5}$$

$$x = \frac{-4 \pm \sqrt{11}}{5}$$

Solution set is $\left\{\frac{-4 \pm \sqrt{11}}{5}\right\}$

(iii) $\sqrt{3}\,x^2 + x = 4\sqrt{3}$ **(Board 2021)** 10301035

Solution: $\sqrt{3}\,x^2 + x = 4\sqrt{3}$

$\sqrt{3}\,x^2 + 1x - 4\sqrt{3} = 0$

As $ax^2 + bx + c = 0$

$\Rightarrow$ $a = \sqrt{3}$, $b = 1$, $c = -4\sqrt{3}$

Using Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4\left(\sqrt{3}\right)\left(-4\sqrt{3}\right)}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm \sqrt{1 + 16\left(\sqrt{3}\right)^2}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm \sqrt{1 + 16(3)}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm \sqrt{1 + 48}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm \sqrt{49}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm 7}{2\sqrt{3}}$$

$$\Rightarrow \quad x = \frac{-1-7}{2\sqrt{3}} \quad \text{or} \quad x = \frac{-1+7}{2\sqrt{3}}$$

$$x = \frac{-\cancel{8}^{4}}{\cancel{2}\sqrt{3}} \quad \text{or} \quad x = \frac{\cancel{6}^{3}}{\cancel{2}\sqrt{3}}$$

$$x = \frac{-4}{\sqrt{3}} \quad \text{or} \quad x = \frac{3}{\sqrt{3}}$$

$$x = \frac{-4}{\sqrt{3}} \quad \text{or} \quad x = \sqrt{3} \qquad \left(\because \frac{a}{\sqrt{a}} = \sqrt{a}\right)$$

Solution set is $\left\{\sqrt{3}, -\frac{4}{\sqrt{3}}\right\}$

(iv) $\mathbf{4x^2 - 14 = 3x}$ **(Board 2022)** 10301036

Solution: $4x^2 - 14 = 3x$

$4x^2 - 3x - 14 = 0$

As $ax^2 + bx + c = 0$

$\Rightarrow$ $a = 4, b = -3, c = -14$

Using Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(-14)}}{2(4)}$$

$$x = \frac{3 \pm \sqrt{9 + 224}}{8}$$

Solution set is $\left\{\frac{3 \pm \sqrt{233}}{8}\right\}$

(v) $\mathbf{6x^2 - 3 - 7x = 0}$ **(Board 2023)** 10301037

Solution: $6x^2 - 3 - 7x = 0$

$6x^2 - 7x - 3 = 0$

As $ax^2 + bx + c = 0$

$\Rightarrow$ $a = 6$, $b = -7$, $c = -3$

Using Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(6)(-3)}}{2(6)}$$

$$x = \frac{7 \pm \sqrt{49 + 72}}{12}$$

$$x = \frac{7 \pm \sqrt{121}}{12}$$

$$x = \frac{7 \pm 11}{12}$$

$\Rightarrow$ $x = \frac{7 - 11}{12}$ or $x = \frac{7 + 11}{12}$

$x = \frac{-4}{12}$ or $x = \frac{18}{12}$

$x = \frac{-1}{3}$ or $x = \frac{3}{2}$

Solution set is $\left\{\frac{-1}{3}, \frac{3}{2}\right\}$

(vi) $\mathbf{3x^2 + 8x + 2 = 0}$ 10301038

Solution: $3x^2 + 8x + 2 = 0$

As $ax^2 + bx + c = 0$

$\Rightarrow$ $a = 3$, $b = 8$, $c = 2$

Using Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(3)(2)}}{2(3)}$$

$$x = \frac{-8 \pm \sqrt{64 - 24}}{6}$$

$$x = \frac{-8 \pm \sqrt{40}}{6}$$

$$x = \frac{-8 \pm \sqrt{4 \times 10}}{6}$$

$$x = \frac{-8 \pm 2\sqrt{10}}{6}$$

$$x = \frac{\cancel{2}\left(-4 \pm \sqrt{10}\right)}{\cancel{6}3}$$

$$x = \frac{-4 \pm \sqrt{10}}{3}$$

Solution set is $\left\{\frac{-4 \pm \sqrt{10}}{3}\right\}$

(vii) $\mathbf{\frac{3}{x-6} - \frac{4}{x-5} = 1}$ 10301039

Solution: $\frac{3}{x-6} - \frac{4}{x-5} = 1$

$$\frac{3(x-5) - 4(x-6)}{(x-6)(x-5)} = 1$$

$3(x-5) - 4(x-6) = (x-6)(x-5)$

$3x - 15 - 4x + 24 = x^2 - 5x - 6x + 30$

$-1x + 9 = x^2 - 11x + 30$

$x^2 - 11x + 1x + 30 - 9 = 0$

$1x^2 - 10x + 21 = 0$

As $ax^2 + bx + c = 0$

$\Rightarrow$ $a = 1$, $b = -10$, $c = 21$

Using quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x=\frac{-(-10)\pm\sqrt{(-10)^2-4(1)(21)}}{2(1)}$$

$$x=\frac{10\pm\sqrt{100-84}}{2}$$

$$x=\frac{10\pm\sqrt{16}}{2}$$

$$x=\frac{10\pm4}{2}$$

$$x=\cancel{2}\frac{(5\pm2)}{\cancel{2}}$$

$\Rightarrow$ $x=5+2$ or $x=5-2$

$x=7$ or $x=3$

Solution set is $\{3, 7\}$

(viii) $\mathbf{\frac{x+2}{x-1}-\frac{4-x}{2x}=2\frac{1}{3}}$ 10301040 **(Board 2023)**

Solution: $\frac{x+2}{x-1}-\frac{4-x}{2x}=2\frac{1}{3}$

$$\frac{2x(x+2)-(4-x)(x-1)}{(x-1)(2x)}=\frac{7}{3}$$

$$\frac{(2x^2+4x)-(4x-4-x^2+x)}{2x^2-2x}=\frac{7}{3}$$

$$\frac{2x^2+4x-5x+4+x^2}{2x^2-2x}=\frac{7}{3}$$

$$\frac{3x^2-x+4}{2x^2-2x}=\frac{7}{3}$$

$$3(3x^2-x+4)=7(2x^2-2x)$$

$$9x^2-3x+12=14x^2-14x$$

$\Rightarrow$ $14x^2-9x^2-14x+3x-12=0$

$$5x^2-11x-12=0$$

As $ax^2+bx+c=0$

$\Rightarrow$ $a=5$, $b=-11$, $c=-12$

Using quadratic formula:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-11)\pm\sqrt{(-11)^2-4(5)(-12)}}{2(5)}$$

$$x=\frac{11\pm\sqrt{121+240}}{10}$$

$$x=\frac{11\pm\sqrt{361}}{10}$$

$$x=\frac{11\pm19}{10}$$

$\Rightarrow$ $x=\frac{11-19}{10}$ or $x=\frac{11+19}{10}$

$x=\frac{-8}{10}$ or $x=\frac{30}{10}$

$x=-\frac{4}{5}$ or $x=3$

Solution set is $\left\{3, \frac{-4}{5}\right\}$

(ix) $\mathbf{\frac{a}{x-b}+\frac{b}{x-a}=2}$ 10301041

Solution: $\frac{a}{x-b}+\frac{b}{x-a}=2$

$$\frac{a(x-a)+b(x-b)}{(x-b)(x-a)}=2$$

$$\frac{ax-a^2+bx-b^2}{x^2-ax-bx+ab}=2$$

$$ax-a^2+bx-b^2=2(x^2-ax-bx+ab)$$

$$ax-a^2+bx-b^2=2x^2-2ax-2bx+2ab$$

$$2x^2-2ax-2bx+2ab-ax+a^2-bx+b^2=0$$

$$2x^2-2ax-ax-2bx-bx+2ab+a^2+b^2=0$$

$$2x^2-3ax-3bx+2ab+a^2+b^2=0$$

$$2x^2-3(a+b)x+(a^2+b^2+2ab)=0$$

$$2x^2-3(a+b)x+(a+b)^2=0$$

As $Ax^2+Bx+C=0$

$\Rightarrow$ $A=2$, $B=-3(a+b)$, $C=(a+b)^2$

Using quadratic formula:

$$x=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

$$x=\frac{-[-3(a+b)]\pm\sqrt{[-3(a+b)]^2-4(2)(a+b)^2}}{2(2)}$$

$$x=\frac{3(a+b)\pm\sqrt{\left[(-3)^2(a+b)^2\right]-8(a+b)^2}}{4}$$

$$x=\frac{3(a+b)\pm\sqrt{9(a+b)^2-8(a+b)^2}}{4}$$

$$x=\frac{3(a+b)\pm\sqrt{(a+b)^2}}{4}$$

$$x=\frac{3(a+b)\pm(a+b)}{4}$$

$x=\frac{3(a+b)-(a+b)}{4}$ or $x=\frac{3(a+b)+(a+b)}{4}$

$x = \frac{2(a+b)}{4}$ or $x = \frac{4(a+b)}{4}$

$x = \frac{a+b}{2}$ or $x = (a+b)$

$x = \frac{1}{2}(a+b)$ or $x = (a+b)$

Solution set is $\left\{(a+b), \frac{1}{2}(a+b)\right\}$

(x) $-(\ell+m) - \ell x^2 + (2\ell+m)x = 0$, $\ell \neq 0$

10301042

Solution: $-(\ell+m) - \ell x^2 + (2\ell+m)x = 0$, $\ell \neq 0$

$\Rightarrow \quad (\ell+m) + \ell x^2 - (2\ell+m)x = 0$

$\Rightarrow \quad \ell x^2 - (2\ell+m)x + (\ell+m) = 0$

As $\quad ax^2 + bx + c = 0$

$\Rightarrow \quad a = \ell$, $b = -(2\ell+m)$, $c = \ell+m$

Using Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-[-(2\ell+m)] \pm \sqrt{[-(2\ell+m)]^2 - 4(\ell)(\ell+m)}}{2(\ell)}$$

$$x = \frac{(2\ell+m) \pm \sqrt{(-1)^2(2\ell+m)^2 - 4\ell(\ell+m)}}{2\ell}$$

$$x = \frac{(2\ell+m) \pm \sqrt{(2\ell)^2 + (m)^2 + 2(2\ell)(m) - 4\ell^2 - 4\ell m}}{2\ell}$$

$$x = \frac{(2\ell+m) \pm \sqrt{\cancel{4\ell^2} + m^2 + \cancel{4\ell m} - \cancel{4\ell^2} - \cancel{4\ell m}}}{2\ell}$$

$$x = \frac{(2\ell+m) \pm \sqrt{m^2}}{2\ell}$$

$$x = \frac{(2\ell+m) \pm m}{2\ell}$$

$x = \frac{2\ell + \cancel{m} - \cancel{m}}{2\ell}$ or $x = \frac{2\ell+m+m}{2\ell}$

$x = \frac{\cancel{2\ell}}{\cancel{2\ell}}$ or $x = \frac{2\ell+2m}{2\ell}$

$x = 1$ or $x = \frac{\cancel{2}(\ell+m)}{\cancel{2}\ell}$

$$x = \frac{\ell+m}{\ell}$$

Solution set is $\left\{1, \frac{\ell+m}{\ell}\right\}$

Equations reducible to quadratic form

Type (i):

The equations of the type $\mathbf{ax^4 + bx^2 + c = 0}$

Example 1:

(Board 2018)

Solve the equation $x^4 - 13x^2 + 36 = 0$ 10301043

Solution: $x^4 - 13x^2 + 36 = 0$ …….. (i)

Let $\quad x^2 = y$ …….. (ii)

Then $x^4 = y^2$

Equation (i) becomes

$$y^2 - 13y + 36 = 0$$
$$y^2 - 9y - 4y + 36 = 0$$
$$y(y-9) - 4(y-9) = 0$$
$$(y-9)(y-4) = 0$$

Either $\quad y - 9 = 0$ or $y - 4 = 0$,

$y = 9$ or $y = 4$

From equation (ii) Put $y = x^2$

$x^2 = 9$ or $x^2 = 4$

$\sqrt{x^2} = \pm\sqrt{9}$ or $\sqrt{x^2} = \pm\sqrt{4}$

$\Rightarrow \quad x = \pm 3$ or $x = \pm 2$

$\therefore \quad$ The solution set is $\{\pm 2, \pm 3\}$

Type (ii):

The equations of the type $\mathbf{ap(x) + \frac{b}{p(x)} = c}$

Example 2:

(Board 2014) 10301044

Solve the equation $2(2x-1) + \frac{3}{2x-1} = 5$

Solution:

$2(2x-1) + \frac{3}{2x-1} = 5$ ……(i)

Let $\quad 2x - 1 = y$ ……(ii)

Then the equation (i) becomes

$$2y + \frac{3}{y} = 5$$

$\frac{2y^2+3}{y} = 5$ (By L.C.M)

or $\quad 2y^2 + 3 = 5y$

$$2y^2 - 5y + 3 = 0$$

Using Quadratic Formula

$$ay^2 + by + c = 0$$

$\Rightarrow \quad a = 2$, $b = -5$, $c = 3$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 2 \times 3}}{2 \times 2}$$

$$y = \frac{5 \pm \sqrt{25-24}}{4} = \frac{5 \pm \sqrt{1}}{4} = \frac{5 \pm 1}{4}$$

We have

$$y=\frac{5+1}{4} \quad \text{or} \quad y=\frac{5-1}{4}$$

$$y=\frac{6}{4}=\frac{3}{2} \quad \text{or} \quad y=\frac{4}{4}=1$$

From equation (ii) putting $(y=2x-1)$

$$2x-1=\frac{3}{2} \quad \text{or} \quad 2x-1=1$$

$$2x=\frac{3}{2}+1 \quad \text{or} \quad 2x=1+1=2$$

$$2x=\frac{5}{2} \quad \text{or} \quad x=\frac{2}{2}$$

$$x=\frac{5}{4} \quad \text{or} \quad x=1$$

Thus, the solution set is $\left\{1,\frac{5}{4}\right\}$

Type (iii): Reciprocal equations of the type:

$$a\left(x^2+\frac{1}{x^2}\right)+b\left(x+\frac{1}{x}\right)+c=0$$

or $ax^4+bx^3+cx^2+bx+a=0$

Q. Define Reciprocal Equation and give example. 10301045

Ans. Reciprocal Equation:

An equation is said to be a reciprocal equation, if it remains unchanged, when x is replaced by $\frac{1}{x}$.

Examples:

(i) $x+\frac{1}{x}=2$ (ii) $ax^4+bx^3+cx^2+bx+a=0$

Q. Show that $ax^4-bx^3+cx^2-bx+a=0$ is reciprocal equation. 10301046

Solution: Replace x by $\frac{1}{x}$ in given equation i.e.

$$ax^4-bx^3+cx^2-bx+a=0$$

$$a\left(\frac{1}{x}\right)^4-b\left(\frac{1}{x}\right)^3+c\left(\frac{1}{x}\right)^2-b\left(\frac{1}{x}\right)+a=0$$

$$\left(\frac{a}{x^4}\right)-\left(\frac{b}{x^3}\right)+\left(\frac{c}{x^2}\right)-\left(\frac{b}{x}\right)+a=0$$

Multiplying both sides by x^4, we get

$$\left(\frac{a}{x^4}\right)x^4-\left(\frac{b}{x^3}\right)x^4+\left(\frac{c}{x^2}\right)x^4-\left(\frac{b}{x}\right)x^4+ax^4=0x^4$$

$$a-bx+cx^2-bx^3+ax^4=0.$$

By arranging these terms we get the same equation .

Thus $ax^4-bx^3+cx^2-bx+a=0$ is a reciprocal equation.

Example 3:

Solve the equation

$$\mathbf{2x^4-5x^3-14x^2-5x+2=0}$$ 10301047

Solution:

$$2x^4-5x^3-14x^2-5x+2=0$$

Dividing each term by x^2.

$$\frac{2x^4}{x^2}-\frac{5x^3}{x^2}-\frac{14x^2}{x^2}-\frac{5x}{x^2}+\frac{2}{x^2}=0$$

$$2x^2-5x-14-\frac{5}{x}+\frac{2}{x^2}=0$$

$$2\left(x^2+\frac{1}{x^2}\right)-5\left(x+\frac{1}{x}\right)-14=0 \quad \text{..... (i)}$$

Let $x+\frac{1}{x}=y$, ... (ii)

Taking square both sides

$$x^2+\frac{1}{x^2}+2(x)\left(\frac{1}{x}\right)=y^2$$

$$x^2+\frac{1}{x^2}=y^2-2$$

So equation (i) becomes

$$2(y^2-2)-5y-14=0$$

$$2y^2-4-5y-14=0$$

$$2y^2-5y-18=0$$

$$2y^2-9y+4y-18=0$$

$$y(2y-9)+2(2y-9)=0$$

$$\Rightarrow \quad (2y-9)(y+2)=0$$

Either $2y-9=0$ or $y+2=0$

From equation (ii) Putting $y=x+\frac{1}{x}$, we have

$$2\left(x+\frac{1}{x}\right)-9=0 \quad \text{or} \quad x+\frac{1}{x}+2=0$$

$$2x+\frac{2}{x}-9=0 \quad \text{or} \quad x^2+1+2x=0$$

$$\boxed{2x^2-9x+2=0} \quad \text{or} \quad \boxed{x^2+2x+1=0}$$

First we solve $2x^2-9x+2=0$ by quadratic formula, we get

$a=2$, $b=-9$, $c=2$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-9)\pm\sqrt{(-9)^2-4\times2\times2}}{2\times2}$$

$$x=\frac{9\pm\sqrt{81-16}}{4}$$

$$x=\frac{9\pm\sqrt{65}}{4}$$

Now we solve $x^2+2x+1=0$ by quadratic formula

$a=1$, $b=2$, $c=1$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-2\pm\sqrt{(2)^2-4\times1\times1}}{2\times1}$$

$$x=\frac{-2\pm\sqrt{4-4}}{2}=\frac{-2\pm0}{2}=\frac{-2}{2}=-1$$

$x=-1$

Thus, the solution set is $\left\{-1,\frac{9\pm\sqrt{65}}{4}\right\}$

Type (iv): Exponential equations:

Q. Define exponential equation and give example. 10301048

Ans. Exponential Equation:

An equation, in which variable occurs in exponent is called exponential equation.

e.g. $2^x-1=0$ or $2^{1+x}+5^{2x}-8=0$

Example 4:

Solve the equation $5^{1+x}+5^{1-x}=26$ 10301049

Solution: $5^{1+x}+5^{1-x}=26$

$5^1.5^x+5^1.5^{-x}=26$

or $5.5^x+\frac{5}{5^x}-26=0$ ……..(i)

Let $5^x=y$. ……..(ii)

Then equation (i) becomes

$5y+\frac{5}{y}-26=0$

$\frac{5y^2+5-26y}{y}=0$

$5y^2+5-26y=0\times y$

$5y^2-26y+5=0$

$5y^2-25y-y+5=0$

$5y(y-5)-1(y-5)=0$

$(y-5)(5y-1)=0$

Either $y-5=0$ or $5y-1=0$,

$y=5$ or $5y=1$

or $y=\frac{1}{5}=5^{-1}$

From equation (ii) Put $y=5^x$

$5^x=5^1$ or $5^x=5^{-1}$

$\Rightarrow x=1$ or $\Rightarrow x=-1$

$\therefore$ The solution set is $\{\pm1\}$.

Type (v): The equations of the type:

$(x+a)(x+b)(x+c)(x+d)=k$

where $a+b=c+d$

Example 5:

Solve the equation

$(x-1)(x+2)(x+8)(x+5)=19$ 10301050

Solution: $(x-1)(x+2)(x+8)(x+5)=19$

Or $[(x-1)(x+8)][(x+2)(x+5)]-19=0$

$(\because -1+8=2+5)$

$(x^2+7x-8)(x^2+7x+10)-19=0$ …. (i)

Let $x^2+7x=y$ …. (ii)

Then equation (i) becomes

$(y-8)(y+10)-19=0$

$y^2+2y-80-19=0$

$y^2+2y-99=0$

$y^2+11y-9y-99=0$

$y(y+11)-9(y+11)=0$

$(y+11)(y-9)=0$

Either $y+11=0$ or $y-9=0$

From equation (ii) Put $y=x^2+7x$

$\boxed{x^2+7x+11=0}$ or $\boxed{x^2+7x-9=0}$

First we solve $x^2+7x+11=0$ by quadratic formula

$a=1$, $b=7$, $c=11$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-7\pm\sqrt{(7)^2-4(1)(11)}}{2(1)}$$

$$x=\frac{-7\pm\sqrt{49-44}}{2}=\frac{-7\pm\sqrt{5}}{2}$$

Now we solve $x^2+7x-9=0$

$a=1$, $b=7$, $c=-9$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-7\pm\sqrt{(7)^2-4(1)(-9)}}{2(1)}$$

$$x=\frac{-7\pm\sqrt{49+36}}{2}$$

$$x=\frac{-7\pm\sqrt{85}}{2}$$

$\therefore$ The solution set is $\left\{\frac{-7\pm\sqrt{5}}{2},\frac{-7\pm\sqrt{85}}{2}\right\}$

EXERCISE 1.3

Q.1 $2x^4 - 11x^2 + 5 = 0$ **(Board 2023)** 10301051

Solution: $2x^4 - 11x^2 + 5 = 0$ ……. (i)

Let $x^2 = y$ ……. (ii)

$(x^2)^2 = y^2$

$x^4 = y^2$

Put $x^2 = y$ and $x^4 = y^2$ in equation (i)

$2x^4 - 11x^2 + 5 = 0$

$2y^2 - 11y + 5 = 0$

$2y^2 - 10y - y + 5 = 0$

$2y(y-5) - 1(y-5) = 0$

$(y-5)(2y-1) = 0$

$y - 5 = 0$ or $2y - 1 = 0$

$y = 0 + 5$ or $2y = 1$

$y = 5$ or $y = \frac{1}{2}$

From equation (ii) Put $y = x^2$

$x^2 = 5$ or $x^2 = \frac{1}{2}$

$\sqrt{x^2} = \pm\sqrt{5}$ or $\sqrt{x^2} = \pm\sqrt{\frac{1}{2}}$

$x = \pm\sqrt{5}$ or $x = \pm\frac{1}{\sqrt{2}}$

The Solution set is $\left\{\pm\frac{1}{\sqrt{2}}, \pm\sqrt{5}\right\}$

Q.2 $2x^4 = 9x^2 - 4$ **(Board 2020,24)** 10301052

Solution: $2x^4 = 9x^2 - 4$

$2x^4 - 9x^2 + 4 = 0$ ……(i)

Let $x^2 = y$ ……(ii)

Taking square on both sides

$(x^2)^2 = y^2$

$x^4 = y^2$

Put $x^2 = y$ and $x^4 = y^2$ in equation (i)

$2y^2 - 9y + 4 = 0$

$2y^2 - 8y - 1y + 4 = 0$

$2y(y-4) - 1(y-4) = 0$

$(y-4)(2y-1) = 0$

$y - 4 = 0$ or $2y - 1 = 0$

$y = 4$ or $2y = 1$

$\Rightarrow$ $y = 4$ or $y = \frac{1}{2}$

From equation (ii) put $y = x^2$

$x^2 = 4$ or $x^2 = \frac{1}{2}$

$\sqrt{x^2} = \pm\sqrt{4}$ or $\sqrt{x^2} = \pm\sqrt{\frac{1}{2}}$

$x = \pm 2$ or $x = \pm\frac{1}{\sqrt{2}}$

The Solution set is $\left\{\pm\frac{1}{\sqrt{2}}, \pm 2\right\}$

Q.3 $5x^{\frac{1}{2}} = 7x^{\frac{1}{4}} - 2$ **(Board 2018)** 10301053

Solution: $5x^{1/2} = 7x^{1/4} - 2$ ……(i)

Let $x^{\frac{1}{4}} = y$ ……(ii)

Taking square on both sides

$\left(x^{\frac{1}{4}}\right)^2 = y^2$

$x^{\frac{1}{2}} = y^2$

Put $x^{\frac{1}{4}} = y$ and $x^{\frac{1}{2}} = y^2$ in equation (i)

$5x^{\frac{1}{2}} = 7x^{\frac{1}{4}} - 2$

$5y^2 = 7y - 2$

$5y^2 - 7y + 2 = 0$

$5y^2 - 5y - 2y + 2 = 0$

$5y(y-1) - 2(y-1) = 0$

$(y-1)(5y-2) = 0$

$y - 1 = 0$ or $5y - 2 = 0$

$y = 1$ or $5y = 2$

$y = 1$ or $y = \frac{2}{5}$

From equation (ii) put $y = x^{\frac{1}{4}}$

$x^{\frac{1}{4}} = 1$ or $x^{\frac{1}{4}} = \frac{2}{5}$

$\left(x^{\frac{1}{4}}\right)^4 = (1)^4$ or $\left(x^{\frac{1}{4}}\right)^4 = \left(\frac{2}{5}\right)^4$

$x = 1$ or $x = \frac{2^4}{5^4} = \frac{16}{625}$

Solution set is $\left\{\frac{16}{625}, 1\right\}$

Q.4 $x^{\frac{2}{3}} + 54 = 15x^{\frac{1}{3}}$ 10301054

Solution: $x^{\frac{2}{3}} + 54 = 15x^{\frac{1}{3}}$ .....(i)

Let $x^{\frac{1}{3}} = y$ .....(ii)

Taking square

$$\left(x^{\frac{1}{3}}\right)^2 = y^2$$

$$x^{\frac{2}{3}} = y^2$$

Put $x^{\frac{1}{3}} = y$ and $x^{\frac{2}{3}} = y^2$ in equation (i)

$$x^{\frac{2}{3}} + 54 = 15x^{\frac{1}{3}}$$
$$y^2 + 54 = 15y$$
$$y^2 - 15y + 54 = 0$$
$$y^2 - 9y - 6y + 54 = 0$$
$$y(y-9) - 6(y-9) = 0$$
$$(y-9)(y-6) = 0$$
$$y - 9 = 0 \quad \text{or} \quad y - 6 = 0$$
$$y = 9 \quad \text{or} \quad y = 6$$

From equation (ii) put $y = x^{\frac{1}{3}}$

$$x^{\frac{1}{3}} = 9 \quad \text{or} \quad x^{\frac{1}{3}} = 6$$
$$\left(x^{\frac{1}{3}}\right)^3 = (9)^3 \quad \text{or} \quad \left(x^{\frac{1}{3}}\right)^3 = (6)^3$$
$$x = 9^3 \quad \text{or} \quad x = 6^3$$
$$x = 9 \times 9 \times 9 \quad \text{or} \quad x = 6 \times 6 \times 6$$
$$x = 729 \quad \text{or} \quad x = 216$$

Solution set is $\{216, 729\}$

Q.5 $3x^{-2} + 5 = 8x^{-1}$ 10301055

Solution: $3x^{-2} + 5 = 8x^{-1}$ .....(i)

Let $x^{-1} = y$ .....(ii)

$$(x^{-1})^2 = y^2$$
$$x^{-2} = y^2$$

Put $x^{-1} = y$ and $x^{-2} = y^2$ in equation (i)

$$3x^{-2} + 5 = 8x^{-1}$$
$$3y^2 + 5 = 8y$$
$$3y^2 - 8y + 5 = 0$$
$$3y^2 - 5y - 3y + 5 = 0$$
$$y(3y-5) - 1(3y-5) = 0$$
$$(3y-5)(y-1) = 0$$
$$3y - 5 = 0 \quad \text{or} \quad y - 1 = 0$$
$$3y = 5 \quad \text{or} \quad y = 1$$
$$y = \frac{5}{3}$$

From equation (ii) put $y = x^{-1}$

$$x^{-1} = \frac{5}{3} \quad \text{or} \quad x^{-1} = 1$$
$$\frac{1}{x} = \frac{5}{3} \quad \text{or} \quad \frac{1}{x} = 1$$
$$x = \frac{3}{5} \quad \text{or} \quad x = 1$$

Solution set is $\left\{\frac{3}{5}, 1\right\}$

Q.6 $(2x^2 + 1) + \dfrac{3}{2x^2 + 1} = 4$ 10301056

Solution: $(2x^2 + 1) + \dfrac{3}{2x^2 + 1} = 4$ .....(i)

Let $2x^2 + 1 = y$ .....(ii)

Replacing $2x^2 + 1$ by "y" in equation (i), we get

$$y + \frac{3}{y} = 4$$

Multiplying both sides by "y"

$$y^2 + 3 = 4y$$
$$y^2 - 4y + 3 = 0$$
$$y^2 - 3y - 1y + 3 = 0$$
$$y(y-3) - 1(y-3) = 0$$
$$(y-3)(y-1) = 0$$
$$y - 3 = 0 \quad \text{or} \quad y - 1 = 0$$
$$y = 3 \quad \text{or} \quad y = 1$$

From equation (ii) Put $y = 2x^2+1$

$$2x^2 + 1 = 3 \quad \text{or} \quad 2x^2 + 1 = 1$$
$$2x^2 = 3 - 1 \quad \text{or} \quad 2x^2 = 1 - 1$$
$$2x^2 = 2 \quad \text{or} \quad 2x^2 = 0$$
$$x^2 = \frac{2}{2} \quad \text{or} \quad x^2 = \frac{0}{2}$$
$$x^2 = 1 \quad \text{or} \quad x^2 = 0$$
$$\sqrt{x^2} = \pm\sqrt{1} \quad \text{or} \quad \sqrt{x^2} = \pm\sqrt{0}$$
$$x = \pm 1 \quad \text{or} \quad x = 0$$

Solution set is $\{-1, 0, 1\}$

Q.7 $\frac{x}{x-3}+4\left(\frac{x-3}{x}\right)=4$ **(Board 2015)** 10301057

Solution: $\frac{x}{x-3}+4\left(\frac{x-3}{x}\right)=4$ ……..(i)

Let $\frac{x}{x-3}=y$ ……..(ii)

$\frac{x-3}{x}=\frac{1}{y}$

Equation (i) becomes

$y+4\left(\frac{1}{y}\right)=4$

Multiplying both sides by "y"

$y^2+4=4y$

$y^2-4y+4=0$

$y^2-2y-2y+4=0$

$y(y-2)-2(y-2)=0$

$(y-2)(y-2)=0$

$(y-2)^2=0$

$y-2=0$

$y=2$

From equation (ii) Put $y=\frac{x}{x-3}$

$\frac{x}{x-3}=2$

$x=2(x-3)$

$x=2x-6$

$6=2x-x$

$6=x \Rightarrow x=6$

Solution set is $\{6\}$

Q.8 $\frac{4x+1}{4x-1}+\frac{4x-1}{4x+1}=2\frac{1}{6}$ 10301058

Solution: $\frac{4x+1}{4x-1}+\frac{4x-1}{4x+1}=2\frac{1}{6}$ …….. (i)

Let $\frac{4x+1}{4x-1}=y$ …….. (ii)

$\frac{4x-1}{4x+1}=\frac{1}{y}$

Equation (i) becomes

$y+\frac{1}{y}=2\frac{1}{6}$

$y+\frac{1}{y}=\frac{13}{6}$

Multiplying both sides by "6y"

$6y^2+6=13y$

$6y^2-13y+6=0$

$6y^2-9y-4y+6=0$

$3y(2y-3)-2(2y-3)=0$

$(2y-3)(3y-2)=0$

$2y-3=0$ or $3y-2=0$

$2y=3$ or $3y=2$

$y=\frac{3}{2}$ or $y=\frac{2}{3}$

From equation (ii) Put $y=\frac{4x+1}{4x-1}$

$\frac{4x+1}{4x-1}=\frac{3}{2}$ or $\frac{4x+1}{4x-1}=\frac{2}{3}$

$2(4x+1)=3(4x-1)$ or $3(4x+1)=2(4x-1)$

$8x+2=12x-3$ or $12x+3=8x-2$

$2+3=12x-8x$ or $12x-8x=-2-3$

$4x=5$ or $4x=-5$

$x=\frac{5}{4}$ or $x=\frac{-5}{4}$

Solution set is $\left\{\pm\frac{5}{4}\right\}$

Q.9 $\frac{x-a}{x+a}-\frac{x+a}{x-a}=\frac{7}{12}$ 10301059

Solution: $\frac{x-a}{x+a}-\frac{x+a}{x-a}=\frac{7}{12}$ ……(i)

Let $\frac{x-a}{x+a}=y$ ……(ii)

Or $\frac{x+a}{x-a}=\frac{1}{y}$

The equation (i) becomes

$y-\frac{1}{y}=\frac{7}{12}$

$\frac{y^2-1}{y}=\frac{7}{12}$

$12(y^2-1)=7y$

$12y^2-12=7y$

$12y^2-7y-12=0$

$12y^2-16y+9y-12=0$

$4y(3y-4)+3(3y-4)=0$

$(3y-4)(4y+3)=0$

$3y-4=0$ or $4y+3=0$

$3y = 4$ or $4y = -3$

$y = \frac{4}{3}$ or $y = \frac{-3}{4}$

From equation (ii) Putting $y = \frac{x-a}{x+a}$, we get

$\frac{x-a}{x+a} = \frac{4}{3}$ or $\frac{x-a}{x+a} = \frac{-3}{4}$

$3(x-a) = 4(x+a)$ or $4(x-a) = -3(x+a)$

$3x - 3a = 4x + 4a$ or $4x - 4a = -3x - 3a$

$-3a - 4a = 4x - 3x$ or $4x + 3x = -3a + 4a$

$-7a = x$ or $7x = a$

$\Rightarrow$ $x = -7a$ or $x = \frac{a}{7}$

The solution set is $\left\{-7a, \frac{a}{7}\right\}$

Q.10 $\mathbf{x^4 - 2x^3 - 2x^2 + 2x + 1 = 0}$ 10301060

Solution: $x^4 - 2x^3 - 2x^2 + 2x + 1 = 0$

Dividing both sides by "x^2"

$$\frac{x^4}{x^2} - \frac{2x^3}{x^2} - \frac{2x^2}{x^2} + \frac{2x}{x^2} + \frac{1}{x^2} = \frac{0}{x^2}$$

$$x^2 - 2x - 2 + \frac{2}{x} + \frac{1}{x^2} = 0$$

$$\left(x^2 + \frac{1}{x^2}\right) - 2x + \frac{2}{x} - 2 = 0$$

$$\left(x^2 + \frac{1}{x^2}\right) - 2\left(x - \frac{1}{x}\right) - 2 = 0 \quad \text{....(i)}$$

Let $x - \frac{1}{x} = y$ .....(ii)

Taking square of both sides

$$\left(x - \frac{1}{x}\right)^2 = y^2$$

$$x^2 + \frac{1}{x^2} - 2 = y^2$$

$$x^2 + \frac{1}{x^2} = y^2 + 2$$

Putting these values in equation (i)

$$y^2 + \cancel{2} - 2(y) - \cancel{2} = 0$$

$$y^2 - 2y = 0$$

$$y(y-2) = 0$$

$y = 0$ or $y - 2 = 0$

$y = 0$ or $y = 2$

From equation (ii) Put $y = x - \frac{1}{x}$

$x - \frac{1}{x} = 0$ or $x - \frac{1}{x} = 2$

$\frac{x^2 - 1}{x} = 0$ or $\frac{x^2 - 1}{x} = 2$

$x^2 - 1 = 0$ or $x^2 - 1 = 2x$

$x^2 = 1$ or $x^2 - 2x - 1 = 0$

$\sqrt{x^2} = \pm\sqrt{1}$ or $\boxed{1x^2 - 2x - 1 = 0}$

$\boxed{x = \pm 1}$

Now Solving $1x^2 - 2x - 1 = 0$ by quadratic formula

$a = 1$, $b = -2$, $c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{+2 \pm \sqrt{4+4}}{2}$$

$$x = \frac{2 \pm \sqrt{8}}{2}$$

$$x = \frac{2 \pm \sqrt{4 \times 2}}{2}$$

$$x = \frac{2 \pm 2\sqrt{2}}{2}$$

$$x = \frac{\cancel{2}\left(1 \pm \sqrt{2}\right)}{\cancel{2}}$$

$$\boxed{x = 1 \pm \sqrt{2}}$$

The solution set is $\left\{\pm 1, 1 \pm \sqrt{2}\right\}$

Q.11 $\mathbf{2x^4 + x^3 - 6x^2 + x + 2 = 0}$ 10301061

Solution: $2x^4 + x^3 - 6x^2 + x + 2 = 0$

Dividing each terms by x^2

$$\frac{2x^4}{x^2} + \frac{x^3}{x^2} - \frac{6\cancel{x^2}}{\cancel{x^2}} + \frac{x}{x^2} + \frac{2}{x^2} = \frac{0}{x^2}$$

$$2x^2 + x - 6 + \frac{1}{x} + \frac{2}{x^2} = 0$$

$$2x^2 + \frac{2}{x^2} + x + \frac{1}{x} - 6 = 0$$

$$2\left(x^2 + \frac{1}{x^2}\right) + \left(x + \frac{1}{x}\right) - 6 = 0 \quad \text{.....(i)}$$

Let $x + \frac{1}{x} = y$ .....(ii)

Taking square of both sides

$$\left(x+\frac{1}{x}\right)^2 = y^2$$

$$x^2+\frac{1}{x^2}+2(\cancel{x})\left(\frac{1}{\cancel{x}}\right) = y^2$$

$$x^2+\frac{1}{x^2}+2 = y^2$$

$$x^2+\frac{1}{x^2} = y^2-2$$

Put $x+\frac{1}{x} = y$ and $x^2+\frac{1}{x^2} = y^2-2$ in equation (i)

$$2\left(x^2+\frac{1}{x^2}\right)+\left(x+\frac{1}{x}\right)-6=0$$

$$2(y^2-2)+y-6=0$$

$$2y^2-4+y-6=0$$

$$2y^2+y-10=0$$

$$2y^2+5y-4y-10=0$$

$$y(2y+5)-2(2y+5)=0$$

$$(2y+5)(y-2)=0$$

$2y+5=0$ or $y-2=0$

$2y=-5$ or $y=2$

$$y=\frac{-5}{2}$$

From equation (ii) Put $y = x+\frac{1}{x}$

$$x+\frac{1}{x}=\frac{-5}{2}$$

$$\frac{x^2+1}{x}=\frac{-5}{2}$$

$$2(x^2+1)=-5x$$

$$2x^2+2=-5x$$

$$2x^2+5x+2=0$$

$$2x^2+4x+1x+2=0$$

$$2x(x+2)+1(x+2)=0$$

$$(x+2)(2x+1)=0$$

$x+2=0$ or $2x+1=0$

$x=-2$ or $2x=-1$

$\boxed{x=-2}$ or $\boxed{x=\frac{-1}{2}}$

$$x+\frac{1}{x}=2$$

$$\frac{x^2+1}{x}=2$$

$$x^2+1=2x$$

$$x^2-2x+1=0$$

$$x^2-1x-1x+1=0$$

$$x(x-1)-1(x-1)=0$$

$$(x-1)(x-1)=0$$

$$(x-1)^2=0$$

$$\sqrt{(x-1)^2}=\sqrt{0}$$

$$x-1=0$$

$$x=1$$

Solution set is $\left\{1,-2,\frac{-1}{2}\right\}$

Q.12 $\mathbf{4.2^{2x+1}-9.2^x+1=0}$ 10301062

Solution: $4.2^{2x+1}-9.2^x+1=0$

$$4.2^{2x}2^1-9.2^x+1=0$$

$$8(2^x)^2-9(2^x)+1=0 \quad \text{..... (i)}$$

Let $2^x = y$ ..... (ii)

The equation (i) becomes

$$8y^2-9y+1=0$$

$$8y^2-8y-1y+1=0$$

$$8y(y-1)-1(y-1)=0$$

$$(y-1)(8y-1)=0$$

$y-1=0$ or $8y-1=0$

$y=1$ or $8y=1$

or $y=\frac{1}{8}$

From equation (ii) Put $y=2^x$

$2^x=1$ or $2^x=\frac{1}{8}$

$2^x=2^o$ or $2^x=\frac{1}{2^3}$

$\Rightarrow$ $x=0$ or $2^x=2^{-3}$

$\Rightarrow$ $x=-3$

Solution set is $\{-3,\ 0\}$

Q.13 $\mathbf{3^{2x+2}=12.3^x-3}$ 10301063

Solution: $3^{2x+2}=12.3^x-3$

$$3^{2x}\ 3^2-12.3^x+3=0$$

$$9(3^x)^2-12.(3^x)+3=0 \quad \text{..... (i)}$$

Let $3^x = y$ ..... (ii)

Put $3^x = y$ in equation (i)

$$9y^2-12y+3=0$$

$$9y^2-9y-3y+3=0$$

$$9y(y-1)-3(y-1)=0$$

$$(y-1)(9y-3)=0$$

$y-1=0$ or $9y-3=0$

$y=1$ or $9y=3$

$\boxed{y=1}$ or $y=\frac{3}{9}$ $\Rightarrow$ $\boxed{y=\frac{1}{3}}$

From equation (ii) Put $y=3^x$

$3^x=1$ or $3^x=\frac{1}{3}$

$3^x=3^0$ or $3^x=3^{-1}$

$\Rightarrow$ $x=0$ or $x=-1$

The solution set is $\{0,-1\}$

Q.14 $2^x + 64.2^{-x} - 20 = 0$ 10301064

Solution: $2^x + 64.2^{-x} - 20 = 0$

$$2^x + \frac{64}{2^x} - 20 = 0 \quad \ldots\ldots\ldots\ldots\text{(i)}$$

Let $2^x = y$ ……(ii)

Put $2^x = y$ in equation (i)

$$y + \frac{64}{y} - 20 = 0$$

Multiply both sides by "y"

$$y^2 + 64 - 20y = 0$$
$$y^2 - 20y + 64 = 0$$
$$y^2 - 16y - 4y + 64 = 0$$
$$y(y-16) - 4(y-16) = 0$$
$$(y-16)\ (y-4) = 0$$
$$y - 16 = 0 \quad \text{or} \quad y - 4 = 0$$
$$y = 16 \quad \text{or} \quad y = 4$$

From equation (ii) put $y = 2^x$

$$2^x = 16 \quad \text{or} \quad 2^x = 4$$
$$2^x = 2^4 \quad \text{or} \quad 2^x = 2^2$$
$$\Rightarrow \quad x = 4 \quad \text{or} \quad x = 2$$

The solution set is { 2, 4 }

Q.15 $(x+1)(x+3)(x-5)(x-7) = 192$ 10301065

Solution: $(x+1)(x+3)(x-5)(x-7) = 192$

$$\boxed{\begin{array}{l} \because \quad a+b = c+d \\ \quad\ \ 1-5 = 3-7 \\ \quad\ \ -4 = -4 \end{array}}$$

$$(x+1)(x-5)(x+3)(x-7) = 192$$
$$(x^2 - 5x + 1x - 5)(x^2 - 7x + 3x - 21) = 192$$
$$(x^2 - 4x - 5)(x^2 - 4x - 21) = 192 \quad \ldots\text{(i)}$$

Let $x^2 - 4x = y$ …….(ii)

The equation (i) becomes

$$(y-5)(y-21) = 192$$
$$y^2 - 21y - 5y + 105 = 192$$
$$y^2 - 26y + 105 - 192 = 0$$
$$y^2 - 26y - 87 = 0$$
$$y^2 - 29y + 3y - 87 = 0$$
$$y(y-29) + 3(y-29) = 0$$
$$(y-29)(y+3) = 0$$
$$y - 29 = 0 \quad \text{or} \quad y + 3 = 0$$
$$y = 29 \quad \text{or} \quad y = -3$$

From equation (ii) Put $y = x^2 - 4x$

$$x^2 - 4x = 29 \quad \text{or} \quad x^2 - 4x = -3$$
$$x^2 - 4x - 29 = 0 \quad \text{or} \quad x^2 - 4x + 3 = 0$$
$$\boxed{x^2 - 4x - 29 = 0} \quad \text{or} \quad \boxed{x^2 - 4x + 3 = 0}$$

First we solve $x^2 - 4x - 29 = 0$ by quadratic formula

$$a = 1 \quad , \quad b = -4 \quad , \quad c = -29$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-29)}}{2(1)}$$
$$x = \frac{4 \pm \sqrt{16 + 116}}{2}$$
$$x = \frac{4 \pm \sqrt{132}}{2}$$
$$x = \frac{4 \pm \sqrt{4 \times 33}}{2}$$
$$x = \frac{4 \pm 2\sqrt{33}}{2}$$
$$x = \frac{\cancel{2}\left(2 \pm \sqrt{33}\right)}{\cancel{2}}$$
$$\boxed{x = 2 \pm \sqrt{33}}$$

Now we solve $x^2 - 4x + 3 = 0$ by factorization

$$x^2 - 4x + 3 = 0$$
$$x^2 - 3x - x + 3 = 0$$
$$x(x-3) - 1(x-3) = 0$$
$$(x-3)(x-1) = 0$$
$$x - 3 = 0 \quad \text{or} \quad x - 1 = 0$$
$$\boxed{x = 3} \quad \text{or} \quad \boxed{x = 1}$$

Solution set is $\left\{1,\ 3,\ 2 \pm \sqrt{33}\right\}$

Q.16 $(x-1)(x-2)(x-8)(x+5) + 360 = 0$ 10301066

Solution: $(x-1)(x-2)(x-8)(x+5) + 360 = 0$

$$\left[(x-1)(x-2)\right]\left[(x-8)(x+5)\right] + 360 = 0$$
$$\left[x^2 - 2x - 1x + 2\right]\left[x^2 + 5x - 8x - 40\right] + 360 = 0$$
$$\left(x^2 - 3x + 2\right)\left(x^2 - 3x - 40\right) + 360 = 0 \quad \ldots\text{(i)}$$

Let $x^2 - 3x = y$ ……(ii)

Put it in equation (i)

$$(y+2)(y-40) + 360 = 0$$
$$y^2 - 40y + 2y - 80 + 360 = 0$$
$$y^2 - 38y + 280 = 0$$
$$y^2 - 28y - 10y + 280 = 0$$

$y(y-28)-10(y-28)=0$

$(y-28)(y-10)=0$

$y-28=0$ or $y-10=0$

$y=28$ or $y=10$

From equation (ii) Put $y = x^2 - 3x$

$x^2-3x=28$	$x^2-3x=10$
$x^2-3x-28=0$	$x^2-3x-10=0$
$x^2-7x+4x-28=0$	$x^2-5x+2x-10=0$
$x(x-7)+4(x-7)=0$	$x(x-5)+2(x-5)=0$
$(x-7)(x+4)=0$	$(x-5)(x+2)=0$
$x-7=0$ or $x+4=0$	$x-5=0$ or $x+2=0$
$x=7$ or $x=-4$	$x=5$ or $x=-2$

The solution set is $\{5, -2, 7, -4\}$

Q. Define Radical Equation. Give example.

Ans. <u>Radical Equation:</u> **(Board 2017,18,24)** 10301067

An equation involving expression of the variable under radical sign is called radical equation. For example $\sqrt{x+1}=2$

(i) The equation of the type $\sqrt{ax+b}=cx+d$

Example 1:

Solve the equation $\sqrt{3x+7}=2x+3$ 10301068

Solution: $\sqrt{3x+7}=2x+3$ ..... (i)

Squaring both sides of the equation (i) , we get

$$\left(\sqrt{3x+7}\right)^2=(2x+3)^2$$

$$3x+7=(2x)^2+2(2x)(3)+(3)^2$$

$$3x+7=4x^2+12x+9$$

$$0=4x^2+12x+9-3x-7$$

$\Rightarrow$ $4x^2+9x+2=0$

By factorization we get,

$$4x^2+8x+x+2=0$$

$$4x(x+2)+1(x+2)=0$$

$$(x+2)(4x+1)=0$$

$$x+2=0 \quad \text{or} \quad 4x+1=0$$

$$x=-2 \quad \text{or} \quad x=\frac{-1}{4}$$

Checking: Putting $x=-\frac{1}{4}$ in the equation (i), we have

$$\sqrt{3\left(-\frac{1}{4}\right)+7}=2\left(-\frac{1}{4}\right)+3$$

$\Rightarrow$ $\sqrt{\frac{-3+28}{4}}=-\frac{1}{2}+3$

$\Rightarrow$ $\sqrt{\frac{25}{4}}=\frac{-1+6}{2}$

$\frac{5}{2}=\frac{5}{2}$ which is true.

Putting $x=-2$ in the equation (i) , we get

$$\sqrt{3(-2)+7}=2(-2)+3$$

$$\sqrt{-6+7}=-4+3$$

$\Rightarrow$ $\sqrt{1}=-1$

$1\neq-1$

On checking, we find that $x=-2$ does not satisfy the equation (i) , so it is an extraneous root. Thus the solution set is $\left\{-\frac{1}{4}\right\}$

(ii) The equation of the type $\sqrt{x+a}+\sqrt{x+b}=\sqrt{x+c}$

Example 2: Solve the equation

$\sqrt{x+3}+\sqrt{x+6}=\sqrt{x+11}$ 10301069

Solution: $\sqrt{x+3}+\sqrt{x+6}=\sqrt{x+11}$ ....(i)

Squaring both sides of the equation (i), we have

$$\left(\sqrt{x+3}+\sqrt{x+6}\right)^2=\left(\sqrt{x+11}\right)^2$$

$$\left(\sqrt{x+3}\right)^2+\left(\sqrt{x+6}\right)^2+2\left(\sqrt{x+3}\right)\left(\sqrt{x+6}\right)=x+11$$

$$x+3+x+6+2\left(\sqrt{x+3}\right)\left(\sqrt{x+6}\right)=x+11$$

$$2x+9+2\sqrt{(x+3)(x+6)}=x+11$$

$$2\sqrt{x^2+6x+3x+18}=x+11-2x-9$$

or $2\sqrt{x^2+9x+18}=-x+2$

Again taking square of both sides we get

$$(2\sqrt{x^2+9x+18})^2=(-x+2)^2$$

$$(2)^2\left(\sqrt{x^2+9x+18}\right)^2=(-x)^2+(2)^2+2(-x)(2)$$

$$4(x^2+9x+18)=x^2+4-4x$$

$$(4x^2+36x+72)=x^2-4x+4$$

$$4x^2+36x+72-x^2+4x-4=0$$

$$3x^2+40x+68=0$$

Applying quadratic formula

$a = 3$, $b = 40$, $c = 68$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-40 \pm \sqrt{(40)^2 - 4(3)(68)}}{2(3)}$$

$$x = \frac{-40 \pm \sqrt{1600 - 816}}{6}$$

$$x = \frac{-40 \pm \sqrt{784}}{6}$$

$$x = \frac{-40 \pm 28}{6}$$

We have

$$x = \frac{-40 + 28}{6} \qquad x = \frac{-40 - 28}{6}$$

$$x = \frac{-12}{6} \qquad x = \frac{-68}{6}$$

$$x = -2 \qquad x = \frac{-34}{3}$$

Checking:

Putting $x = -2$ in (i)

$$\sqrt{-2+3} + \sqrt{-2+6} = \sqrt{-2+11}$$

$$\sqrt{1} + \sqrt{4} = \sqrt{9}$$

$$1 + 2 = 3$$

$3 = 3$ which is true

So one root is 3

Now Putting $x = \frac{-34}{3}$ in the equation (i), we have

$$\sqrt{\frac{-34}{3} + 3} + \sqrt{\frac{-34}{3} + 6} = \sqrt{\frac{-34}{3} + 11}$$

or

$$\sqrt{\frac{-34+9}{3}} + \sqrt{\frac{-34+18}{3}} = \sqrt{\frac{-34+33}{3}}$$

$$\Rightarrow \quad \sqrt{\frac{25}{3} \times (-1)} + \sqrt{\frac{16}{3} \times (-1)} = \sqrt{\frac{1}{3} \times (-1)}$$

$$\Rightarrow \quad \frac{5}{\sqrt{3}} i + \frac{4}{\sqrt{3}} i = \frac{1}{\sqrt{3}} i \qquad \left(\because i = \sqrt{-1}\right)$$

$\frac{9}{\sqrt{3}} i = \frac{1}{\sqrt{3}} i$ which is not true.

As $\frac{-34}{3}$ is an extraneous root,

So, the solution set is $\{-2\}$.

(iii) The equation of the type $\sqrt{x^2 + px + m} + \sqrt{x^2 + px + n} = q$

Example 3:

Solve the equation **(Board 2014)**

$$\sqrt{x^2 - 3x + 36} - \sqrt{x^2 - 3x + 9} = 3 \qquad 10301070$$

Solution: $\sqrt{x^2 - 3x + 36} - \sqrt{x^2 - 3x + 9} = 3$ ...(i)

Let $x^2 - 3x = y$ .....(ii)

Put it in equation (i)

Then $\sqrt{y + 36} - \sqrt{y + 9} = 3$

Squaring both sides

$$\left(\sqrt{y+36} - \sqrt{y+9}\right)^2 = (3)^2$$

$$\left(\sqrt{y+36}\right)^2 + \left(\sqrt{y+9}\right)^2 - 2\left(\sqrt{y+36}\right)\left(\sqrt{y+9}\right) = 9$$

$$y + 36 + y + 9 - 2\sqrt{(y+36)(y+9)} = 9$$

$$2y + 45 - 2\sqrt{(y^2 + 36y + 9y + 324)} = 9$$

$$-2\sqrt{(y^2 + 45y + 324)} = 9 - 2y - 45$$

$$-2\sqrt{y^2 + 45y + 324} = -2y - 36$$

$$\cancel{-2}\sqrt{y^2 + 45y + 324} = \cancel{-2}(y + 18)$$

Again squaring both sides

$$\left(\sqrt{y^2 + 45y + 324}\right)^2 = (y + 18)^2$$

$$y^2 + 45y + 324 = (y)^2 + 2(y)(18) + (18)^2$$

$$\cancel{y^2} + 45y + \cancel{324} = \cancel{y^2} + 36y + \cancel{324}$$

$$45y - 36y = 0$$

$$9y = 0$$

$$\Rightarrow \quad y = 0$$

From equation (ii) put $y = x^2 - 3x$ so

$$x^2 - 3x = 0$$

$$\Rightarrow \quad x(x - 3) = 0$$

Either $x = 0$ or $x - 3 = 0$

or $x = 3$

$\therefore$ $x = 0, 3$ are the roots of the equation.

Thus, the solution set is $\{0, 3\}$

EXERCISE 1.4

Solve the following equations:

Q.1 $2x+5=\sqrt{7x+16}$ **(Board 2017)** 10301071

Solution: $2x+5=\sqrt{7x+16}$ .....(i)

Squaring both sides

$$(2x+5)^2=\left(\sqrt{7x+16}\right)^2$$
$$(2x)^2+(5)^2+2(2x)(5)=7x+16$$
$$4x^2+25+20x=7x+16$$
$$4x^2+20x-7x+25-16=0$$
$$4x^2+13x+9=0$$
$$4x^2+4x+9x+9=0$$
$$4x(x+1)+9(x+1)=0$$
$$(x+1)(4x+9)=0$$
$$x+1=0 \quad \text{or} \quad 4x+9=0$$
$$x=-1 \quad \text{or} \quad 4x=-9$$
$$x=-1 \quad \text{or} \quad x=\frac{-9}{4}$$

Checking:

Putting $x=-1$ in the equation (i) we have

$$2x+5=\sqrt{7x+16}$$
$$2(-1)+5=\sqrt{7(-1)+16}$$
$$-2+5=\sqrt{-7+16}$$
$$3=\sqrt{9}$$
$$3=3 \quad \text{which is true}$$

Putting $x=\frac{-9}{4}$ in the Equation (i) we have

$$2x+5=\sqrt{7x+16}$$
$$2\left(\frac{-9}{4}\right)+5=\sqrt{7\left(\frac{-9}{4}\right)+16}$$
$$\frac{-18}{4}+5=\sqrt{\frac{-63}{4}+16}$$
$$\frac{-18+20}{4}=\sqrt{\frac{-63+64}{4}}$$
$$\frac{2}{4}=\sqrt{\frac{1}{4}} \Rightarrow \frac{1}{2}=\frac{1}{2} \text{ which is true}$$

So, the solution set is $\left\{-1, \frac{-9}{4}\right\}$

Q.2 $\sqrt{x+3}=3x-1$ **(Board 2014)** 10301072

Solution: $\sqrt{x+3}=3x-1$ .....(i)

Squaring both sides

$$\left(\sqrt{x+3}\right)^2=(3x-1)^2$$
$$x+3=(3x)^2+(1)^2-2(3x)(1)$$
$$x+3=9x^2+1-6x$$
$$9x^2-6x-x+1-3=0$$
$$9x^2-7x-2=0$$
$$9x^2-9x+2x-2=0$$
$$9x(x-1)+2(x-1)=0$$
$$(x-1)(9x+2)=0$$
$$x-1=0 \quad \text{or} \quad 9x+2=0$$
$$x=1 \quad \text{or} \quad 9x=-2$$
$$\text{or} \quad x=\frac{-2}{9}$$

Checking:

Putting x = 1 in the equation (i) we have

$$\sqrt{x+3}=3x-1$$
$$\sqrt{1+3}=3(1)-1$$
$$\sqrt{4}=3-1$$
$$2=2 \quad \text{which is true}$$

Putting $x=\frac{-2}{9}$ in the equation (i) we have

$$\sqrt{x+3}=3x-1$$
$$\sqrt{\frac{-2}{9}+3}=3\left(\frac{-2}{9}\right)-1$$
$$\sqrt{\frac{-2+27}{9}}=\frac{-6}{9}-1$$
$$\sqrt{\frac{25}{9}}=\frac{-6-9}{9}$$
$$\frac{5}{3}=\frac{-\cancel{15}^{5}}{\cancel{9}_{3}}$$
$$\frac{5}{3}=\frac{-5}{3} \quad \text{which is not true}$$

As $\frac{-2}{9}$ is an extraneous root

So, the solution set is $\{1\}$

Q.3 $\mathbf{4x = \sqrt{13x+14} - 3}$ 10301073

Solution: $4x = \sqrt{13x+14} - 3$ .....(i)

$4x + 3 = \sqrt{13x+14}$

Squaring both sides

$(4x+3)^2 = \left(\sqrt{13x+14}\right)^2$

$(4x)^2 + (3)^2 + 2(4x)(3) = 13x + 14$

$16x^2 + 9 + 24x = 13x + 14$

$16x^2 + 9 + 24x - 13x - 14 = 0$

$16x^2 + 11x - 5 = 0$

$16x^2 + 16x - 5x - 5 = 0$

$16x(x+1) - 5(x+1) = 0$

$(x+1)(16x-5) = 0$

$x + 1 = 0$ or $16x - 5 = 0$

$x = -1$ or $16x = 5$

$\boxed{x = -1}$ or $\boxed{x = \frac{5}{16}}$

Checking:

Putting $x = -1$ in the equation (i) we have

$4x = \sqrt{13x+14} - 3$

$4(-1) = \sqrt{13(-1)+14} - 3$

$-4 = \sqrt{-13+14} - 3$

$-4 = \sqrt{1} - 3$

$-4 = 1 - 3$

$-4 \neq -2$

So –1 is an extraneous root

Putting $x = \frac{5}{16}$ in the equation (i) we have

$4x = \sqrt{13x+14} - 3$

$4\left(\frac{5}{16}\right) = \sqrt{13\left(\frac{5}{16}\right)+14} - 3$

$\frac{20}{16} = \sqrt{\frac{65}{16}+14} - 3$

$\frac{5}{4} = \sqrt{\frac{65+224}{16}} - 3$

$\frac{5}{4} = \sqrt{\frac{289}{16}} - 3$

$\frac{5}{4} = \frac{17}{4} - 3$

$\frac{5}{4} = \frac{17-12}{4}$

$\frac{5}{4} = \frac{5}{4}$ which is true

So, the solution set is $\left\{\frac{5}{16}\right\}$

Q.4 $\mathbf{\sqrt{3x+100} - x = 4}$ **(Board 2022)** 10301074

Solution: $\mathbf{\sqrt{3x+100} - x = 4}$**.....(i)**

$\sqrt{3x+100} = 4 + x$

Squaring both sides

$\left(\sqrt{3x+100}\right)^2 = (4+x)^2$

$3x + 100 = (4)^2 + (x)^2 + 2(4)(x)$

$3x + 100 = 16 + x^2 + 8x$

$x^2 + 8x + 16 - 3x - 100 = 0$

$x^2 + 5x - 84 = 0$

$x^2 + 12x - 7x - 84 = 0$

$x(x+12) - 7(x+12) = 0$

$(x+12)(x-7) = 0$

$x + 12 = 0$ or $x - 7 = 0$

$x = -12$ or $x = 7$

Checking:

Putting $x = -12$ in the equation (i) we have

$\sqrt{3x+100} - x = 4$

$\sqrt{3(-12)+100} - (-12) = 4$

$\sqrt{-36+100} + 12 = 4$

$\sqrt{64} + 12 = 4$

$8 + 12 = 4$

$20 \neq 4$

So –12 is an extraneous root

Putting $x = 7$ in the equation (i) we have

$\sqrt{3x+100} - x = 4$

$\sqrt{3(7)+100} - 7 = 4$

$\sqrt{21+100} - 7 = 4$

$\sqrt{121} - 7 = 4$

$11 - 7 = 4$

$4 = 4$ which is true

So, the solution set is $\{7\}$

Q.5 $\mathbf{\sqrt{x+5} + \sqrt{x+21} = \sqrt{x+60}}$ 10301075

Solution: $\sqrt{x+5} + \sqrt{x+21} = \sqrt{x+60}$ …(i)

Squaring on both sides

$\left(\sqrt{x+5} + \sqrt{x+21}\right)^2 = \left(\sqrt{x+60}\right)^2$

$\left(\sqrt{x+5}\right)^2 + \left(\sqrt{x+21}\right)^2 + 2\left(\sqrt{x+5}\right)\left(\sqrt{x+21}\right) = x + 60$

$x + 5 + x + 21 + 2\left(\sqrt{x+5}\right)\left(\sqrt{x+21}\right) = x + 60$

$2x + 26 + 2\sqrt{x^2 + 21x + 5x + 105} = x + 60$

$$2\sqrt{x^2+26x+105} = x+60-2x-26$$
$$2\sqrt{x^2+26x+105} = -x+34$$
$$2\sqrt{x^2+26x+105} = 34-x$$

Again squaring both sides

$$\left(2\sqrt{x^2+26x+105}\right)^2 = (34-x)^2$$
$$4(x^2+26x+105) = (34)^2 + x^2 - 2(34)(x)$$
$$4x^2+104x+420 = 1156 + x^2 - 68x$$
$$4x^2+104x+420-1156-x^2+68x = 0$$
$$3x^2+172x-736 = 0$$
$$ax^2 + bx + c = 0$$
$$a = 3\ ,\ b = 172\ ,\ c = -736$$

By Using Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$x = \frac{-172 \pm \sqrt{(172)^2 - 4(3)(-736)}}{2(3)}$$
$$x = \frac{-172 \pm \sqrt{29584+8832}}{6}$$
$$x = \frac{-172 \pm \sqrt{38416}}{6}$$
$$x = \frac{-172 \pm 196}{6}$$
$$x = \frac{-172-196}{6} \text{ or } x = \frac{-172+196}{6}$$
$$x = \frac{-368}{6} \quad \text{or} \quad x = \frac{24}{6}$$
$$x = \frac{-184}{3} \quad \text{or} \quad x = 4$$

Checking:

Putting $x = 4$ in equation (i)

$$\sqrt{x+5} + \sqrt{x+21} = \sqrt{x+60}$$
$$\sqrt{4+5} + \sqrt{4+21} = \sqrt{4+60}$$
$$\sqrt{9} + \sqrt{25} = \sqrt{64}$$
$$3+5 = 8$$
$$8 = 8 \qquad \text{which is true}$$

Putting $x = \frac{-184}{3}$

$$\sqrt{x+5} + \sqrt{x+21} = \sqrt{x+60}$$
$$\sqrt{\frac{-184}{3}+5} + \sqrt{\frac{-184}{3}+21} = \sqrt{\frac{-184}{3}+60}$$
$$\sqrt{\frac{-184+15}{3}} + \sqrt{\frac{-184+63}{3}} = \sqrt{\frac{-184+180}{3}}$$
$$\sqrt{\frac{-169}{3}} + \sqrt{\frac{-121}{3}} = \sqrt{\frac{-4}{3}}$$
$$\sqrt{\frac{-1\times169}{3}} + \sqrt{\frac{-1\times121}{3}} = \sqrt{\frac{-1\times4}{3}}$$
$$\frac{13i}{\sqrt{3}} + \frac{11i}{\sqrt{3}} = \frac{2i}{\sqrt{3}} \qquad \because i = \sqrt{-1}$$
$$\frac{13i+11i}{\sqrt{3}} = \frac{2i}{\sqrt{3}}$$
$$\frac{24i}{\sqrt{3}} = \frac{2i}{\sqrt{3}} \qquad \text{Which is not true}$$

As $x = \frac{-184}{3}$ is extraneous root

so solution set is $\{4\}$

Q.6 $\sqrt{x+1} + \sqrt{x-2} = \sqrt{x+6}$ 10301076

Solution: $\sqrt{x+1} + \sqrt{x-2} = \sqrt{x+6}$ .....(i)

Squaring both sides

$$\left(\sqrt{x+1} + \sqrt{x-2}\right)^2 = \left(\sqrt{x+6}\right)^2$$
$$\left(\sqrt{x+1}\right)^2 + \left(\sqrt{x-2}\right)^2 + 2\left(\sqrt{x+1}\right)\left(\sqrt{x-2}\right) = x+6$$
$$x+1+x-2+2\sqrt{(x+1)(x-2)} = x+6$$
$$2x-1+2\sqrt{x^2-2x+1x-2} = x+6$$
$$2\sqrt{x^2-x-2} = x+6-2x+1$$
$$2\sqrt{x^2-x-2} = 7-x$$

Again squaring both sides

$$\left(2\sqrt{x^2-x-2}\right)^2 = (7-x)^2$$
$$4(x^2-x-2) = (7)^2 + (x)^2 - 2(7)(x)$$
$$4x^2-4x-8 = 49 + x^2 - 14x$$
$$4x^2 - x^2 - 4x + 14x - 8 - 49 = 0$$
$$3x^2 + 10x - 57 = 0$$
$$3x^2 + 19x - 9x - 57 = 0$$
$$x(3x+19) - 3(3x+19) = 0$$
$$(3x+19)(x-3) = 0$$
$$3x+19 = 0 \quad \text{or} \quad x-3 = 0$$
$$3x = -19 \quad \text{or} \quad x = 3$$
$$x = \frac{-19}{3}$$

Checking:

Putting $x=3$ in the equation (i)

$$\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6}$$
$$\sqrt{3+1}+\sqrt{3-2}=\sqrt{3+6}$$
$$\sqrt{4}+\sqrt{1}=\sqrt{9}$$
$$2+1=3$$
$$3=3 \qquad \text{which is true}$$

Putting $x=\frac{-19}{3}$ in the equation (i)

$$\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6}$$
$$\sqrt{\frac{-19}{3}+1}+\sqrt{\frac{-19}{3}-2}=\sqrt{\frac{-19}{3}+6}$$
$$\sqrt{\frac{-19+3}{3}}+\sqrt{\frac{-19-6}{3}}=\sqrt{\frac{-19+18}{3}}$$
$$\sqrt{\frac{-1\times16}{3}}+\sqrt{\frac{-1\times25}{3}}=\sqrt{\frac{-1\times1}{3}}$$
$$\frac{4i}{\sqrt{3}}+\frac{5i}{\sqrt{3}}=\frac{1i}{\sqrt{3}} \qquad \because i=\sqrt{-1}$$
$$\frac{4i+5i}{\sqrt{3}}=\frac{1i}{\sqrt{3}}$$
$$\frac{9i}{\sqrt{3}}=\frac{1i}{\sqrt{3}} \qquad \text{which is not true}$$

As $x=\frac{-19}{3}$ is an extraneous root

So the solution set is $\{3\}$

Q.7 $\sqrt{11-x}-\sqrt{6-x}=\sqrt{27-x}$ 10301077

Solution: $\sqrt{11-x}-\sqrt{6-x}=\sqrt{27-x}$ .....(i)

Squaring both sides

$$\left(\sqrt{11-x}-\sqrt{6-x}\right)^2=\left(\sqrt{27-x}\right)^2$$
$$\left(\sqrt{11-x}\right)^2+\left(\sqrt{6-x}\right)^2-2\left(\sqrt{11-x}\right)\left(\sqrt{6-x}\right)=27-x$$
$$11-x+6-x-2\sqrt{(11-x)(6-x)}=27-x$$
$$17-2x-2\sqrt{66-11x-6x+x^2}=27-x$$
$$-2\sqrt{66-17x+x^2}=27-x-17+2x$$
$$-2\sqrt{66-17x+x^2}=10+x$$

Again squaring both sides

$$\left(-2\sqrt{66-17x+x^2}\right)^2=(10+x)^2$$
$$(-2)^2\left(\sqrt{66-17x+x^2}\right)^2=(10+x)^2$$
$$4(66-17x+x^2)=(10)^2+(x)^2+2(10)(x)$$
$$264-68x+4x^2=100+x^2+20x$$
$$4x^2-x^2-68x-20x+264-100=0$$
$$3x^2-88x+164=0$$

By applying quadratic formula

$$a=3 \quad , \quad b=-88 \quad , \quad c=164$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-88)\pm\sqrt{(-88)^2-4(3)(164)}}{2(3)}$$
$$x=\frac{88\pm\sqrt{7744-1968}}{6}$$
$$x=\frac{88\pm\sqrt{5776}}{6}$$
$$x=\frac{88-76}{6} \quad \text{or} \quad x=\frac{88+76}{6}$$
$$x=\frac{\cancel{12}^{\,2}}{\cancel{6}} \quad \text{or} \quad x=\frac{\cancel{164}^{\,82}}{\cancel{6}_{\,3}}$$
$$x=2 \quad \text{or} \quad x=\frac{82}{3}$$

Checking:

Putting $x=2$ in the equation (i)

$$\sqrt{11-x}-\sqrt{6-x}=\sqrt{27-x}$$
$$\sqrt{11-2}-\sqrt{6-2}=\sqrt{27-2}$$
$$\sqrt{9}-\sqrt{4}=\sqrt{25}$$
$$3-2=5$$
$$1=5 \qquad \text{which is not true}$$

Putting $x=\frac{82}{3}$ in the equation (i)

$$\sqrt{11-x}-\sqrt{6-x}=\sqrt{27-x}$$
$$\sqrt{11-\frac{82}{3}}-\sqrt{6-\frac{82}{3}}=\sqrt{27-\frac{82}{3}}$$
$$\sqrt{\frac{33-82}{3}}-\sqrt{\frac{18-82}{3}}=\sqrt{\frac{81-82}{3}}$$
$$\sqrt{\frac{-1\times49}{3}}-\sqrt{\frac{-1\times64}{3}}=\sqrt{\frac{-1\times1}{3}}$$
$$\frac{7i}{\sqrt{3}}-\frac{8i}{\sqrt{3}}=\frac{i}{\sqrt{3}} \qquad \because i=\sqrt{-1}$$
$$\frac{7i-8i}{\sqrt{3}}=\frac{i}{\sqrt{3}}$$
$$\frac{-i}{\sqrt{3}}=\frac{i}{\sqrt{3}} \qquad \text{which is not true}$$

Here 2 and $\frac{82}{3}$ are extraneous roots.

So, the solution set is $\{\ \}$

Q.8 $\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$ 10301078

Solution: $\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$ ……..(i)

Squaring both sides

$$\left(\sqrt{4a+x}-\sqrt{a-x}\right)^2=\left(\sqrt{a}\right)^2$$

$$\left(\sqrt{4a+x}\right)^2+\left(\sqrt{a-x}\right)^2-2\left(\sqrt{4a+x}\right)\left(\sqrt{a-x}\right)=a$$

$$4a+\cancel{x}+a-\cancel{x}-2\sqrt{(4a+x)(a-x)}=a$$

$$5a-2\sqrt{4a^2-4ax+ax-x^2}=a$$

$$-2\sqrt{4a^2-3ax-x^2}=a-5a$$

$$-2\sqrt{4a^2-3ax-x^2}=-4a$$

$$\sqrt{4a^2-3ax-x^2}=\frac{-\cancel{4}^{2}a}{-\cancel{2}}$$

$$\sqrt{4a^2-3ax-x^2}=2a$$

Again squaring both sides

$$\left(\sqrt{4a^2-3ax-x^2}\right)^2=(2a)^2$$

$$4a^2-3ax-x^2=4a^2$$

$$4a^2-3ax-x^2=4a^2$$

$$4\cancel{a^2}-4\cancel{a^2}=3ax+x^2$$

$\Rightarrow$ $3ax+x^2=0$

$x(3a+x)=0$

$x=0$ or $3a+x=0$

$x=0$ or $x=-3a$

Checking:

Putting $x=0$ in the equation (i)

$$\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$$

$$\sqrt{4a+0}-\sqrt{a-0}=\sqrt{a}$$

$$\sqrt{4a}-\sqrt{a}=\sqrt{a}$$

$$2\sqrt{a}-\sqrt{a}=\sqrt{a}$$

$\sqrt{a}=\sqrt{a}$ which is true

Putting $x=-3a$ in the equation (i)

$$\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$$

$$\sqrt{4a-3a}-\sqrt{a+3a}=\sqrt{a}$$

$$\sqrt{a}-\sqrt{4a}=\sqrt{a}$$

$$\sqrt{a}-2\sqrt{a}=\sqrt{a}$$

$-\sqrt{a}=\sqrt{a}$ which is not true

As $-3a$ is extraneous root

So, the solution set is $\{0\}$

Q.9 $\sqrt{x^2+x+1}-\sqrt{x^2+x-1}=1$ 10301079

Solution: $\sqrt{x^2+x+1}-\sqrt{x^2+x-1}=1$……(i)

Let $x^2+x=y$ …….(ii)

Put it in equation (i)

$$\sqrt{y+1}-\sqrt{y-1}=1$$

Squaring both sides

$$\left(\sqrt{y+1}-\sqrt{y-1}\right)^2=(1)^2$$

$$\left(\sqrt{y+1}\right)^2+\left(\sqrt{y-1}\right)^2-2\left(\sqrt{y+1}\right)\left(\sqrt{y-1}\right)=1$$

$$y+\cancel{1}+y-\cancel{1}-2\sqrt{(y+1)(y-1)}=1$$

$$2y-2\sqrt{y^2-1}=1$$

$\Rightarrow$ $2y-1=2\sqrt{y^2-1}$

Again squaring both sides

$$(2y-1)^2=\left(2\sqrt{y^2-1}\right)^2$$

$$(2y)^2+(1)^2-2(2y)(1)=4\left(y^2-1\right)$$

$$4y^2+1-4y=4y^2-4$$

$$4\cancel{y^2}+1-4y=4\cancel{y^2}-4$$

$$1-4y=-4$$

$$1+4=4y$$

$$5=4y$$

$\Rightarrow$ $4y=5$

From equation (ii) Put $y=x^2+x$

$$4y=5$$

$$4(x^2+x)=5$$

$$4x^2+4x=5$$

$$4x^2+4x-5=0$$

$$ax^2+bx+c=0$$

Solving by quadratic formula

$a=4$, $b=4$, $c=-5$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-4\pm\sqrt{(4)^2-4(4)(-5)}}{2(4)}$$

$$x=\frac{-4\pm\sqrt{16+80}}{8}$$

$$x=\frac{-4\pm\sqrt{96}}{8}$$

$$x=\frac{-4\pm\sqrt{16\times6}}{8}$$

$$x = \frac{-4 \pm 4\sqrt{6}}{8}$$

$$x = \frac{\cancel{4}\left(-1 \pm \sqrt{6}\right)}{\cancel{8}_{2}}$$

$$x = \frac{-1 \pm \sqrt{6}}{2}$$

On checking we know

The solution set is $\left\{\frac{-1 \pm \sqrt{6}}{2}\right\}$

Q.10 $\sqrt{x^2 + 3x + 8} + \sqrt{x^2 + 3x + 2} = 3$ 10301080

Solution: $\sqrt{x^2 + 3x + 8} + \sqrt{x^2 + 3x + 2} = 3$ ....(i)

Let $x^2 + 3x = y$ .....(ii)

Put it in equation (i)

$$\sqrt{y+8} + \sqrt{y+2} = 3$$

Squaring both sides

$$\left(\sqrt{y+8} + \sqrt{y+2}\right)^2 = (3)^2$$

$$\left(\sqrt{y+8}\right)^2 + \left(\sqrt{y+2}\right)^2 + 2\left(\sqrt{y+8}\right)\left(\sqrt{y+2}\right) = 9$$

$$y + 8 + y + 2 + 2\sqrt{y^2 + 2y + 8y + 16} = 9$$

$$2y + 10 + 2\sqrt{y^2 + 10y + 16} = 9$$

$$2\sqrt{y^2 + 10y + 16} = 9 - 2y - 10$$

$$2\sqrt{y^2 + 10y + 16} = -2y - 1$$

$$2\sqrt{y^2 + 10y + 16} = -(2y + 1)$$

Again squaring on both sides

$$\left(2\sqrt{y^2 + 10y + 16}\right)^2 = [-(2y+1)]^2$$

$$4(y^2 + 10y + 16) = (2y+1)^2$$

$$4y^2 + 40y + 64 = (2y)^2 + (1)^2 + 2(2y)(1)$$

$$\cancel{4y^2} + 40y + 64 = \cancel{4y^2} + 1 + 4y$$

$$40y + 64 - 1 - 4y = 0$$

$$36y + 63 = 0$$

$$9(4y + 7) = 0$$

$$\Rightarrow \quad 4y + 7 = 0 \qquad (\because 9 \neq 0)$$

Putting the value of y from eq. (ii)

$$4\left(x^2 + 3x\right) + 7 = 0$$

$$4x^2 + 12x + 7 = 0$$

$$ax^2 + bx + c = 0$$

Solving by quadratic formula

$$a = 4,\ b = 12,\ c = 7$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-12 \pm \sqrt{(12)^2 - 4(4)(7)}}{2(4)}$$

$$x = \frac{-12 \pm \sqrt{144 - 112}}{8}$$

$$x = \frac{-12 \pm \sqrt{32}}{8}$$

$$x = \frac{-12 \pm \sqrt{16 \times 2}}{8}$$

$$x = \frac{-12 \pm 4\sqrt{2}}{8}$$

$$x = \frac{{}^{1}\cancel{4}\left(-3 \pm \sqrt{2}\right)}{\cancel{8}_{2}}$$

On checking we know

the solution set is $\left\{\frac{-3 \pm \sqrt{2}}{2}\right\}$

Q.11 $\sqrt{x^2 + 3x + 9} + \sqrt{x^2 + 3x + 4} = 5$ 10301081

Solution: $\sqrt{x^2 + 3x + 9} + \sqrt{x^2 + 3x + 4} = 5$ ....(i)

Let $x^2 + 3x = y$ .....(ii)

Put it in equation (i)

$$\sqrt{y+9} + \sqrt{y+4} = 5$$

Squaring both sides

$$\left(\sqrt{y+9} + \sqrt{y+4}\right)^2 = 5^2$$

$$\left(\sqrt{y+9}\right)^2 + \left(\sqrt{y+4}\right)^2 + 2\left(\sqrt{y+9}\right)\left(\sqrt{y+4}\right) = 25$$

$$y + 9 + y + 4 + 2\sqrt{(y+9)(y+4)} = 25$$

$$2y + 13 + 2\sqrt{y^2 + 4y + 9y + 36} = 25$$

$$2\sqrt{y^2 + 13y + 36} = 25 - 2y - 13$$

$$2\sqrt{y^2 + 13y + 36} = 12 - 2y$$

$$\cancel{2}\sqrt{y^2 + 13y + 36} = \cancel{2}(6 - y)$$

$$\sqrt{y^2 + 13y + 36} = 6 - y$$

Again taking squaring both sides

$$\left(\sqrt{y^2 + 13y + 36}\right)^2 = (6 - y)^2$$

$$y^2+13y+36=(6)^2+(y)^2-2(6)(y)$$
$$y^2+13y+36=36+y^2-12y$$
$$\cancel{y^2}+13y+\cancel{36}-\cancel{36}-\cancel{y^2}+12y=0$$
$$13y+12y=0$$
$$25y=0$$
$$y=\frac{0}{25}$$
$$y=0$$

From equation (ii) Put $x^2+3x=y$

$$x^2+3x=0$$
$$x(x+3)=0$$
$$x=0 \quad \text{or} \quad x+3=0$$
$$x=-3$$

On checking we know

the solution set is $\{-3, 0\}$

MISCELLANEOUS EXERCISE – 1

Q. 1 Multiple Choice Questions

Four possible answers are given for the following questions. Tick (✓) the correct answer.

1. Standard form of quadratic equation is: **(Board 2014,23)** 10301082
(a) bx+c= 0, b≠0
(b) ax^2+bx+c=0, a≠0
(c) ax^2=bx, a≠0
(d) ax^2=0, a≠0

2. The number of terms in a standard quadratic equation ax^2+bx+c= 0 is: **(Board 2015,18,24)** 10301083
(a) 1 (b) 2
(c) 3 (d) 4

3. The number of methods to solve a quadratic equation is: **(Board 2020)** 10301084
(a) 1 (b) 2
(c) 3 (d) 4

4. The quadratic formula is: 10301085
(a) $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ (b) $\frac{b\pm\sqrt{b^2-4ac}}{2a}$
(c) $\frac{-b\pm\sqrt{b^2+4ac}}{2a}$ (d) $\frac{b\pm\sqrt{b^2+4ac}}{2a}$

5. Two linear factors of $x^2-15x+56$ are:
(a) $(x-7)$ and $(x+8)$ 10301086
(b) $(x+7)$ and $(x-8)$ **(Board 2014,15,20,21)**
(c) $(x-7)$ and $(x-8)$
(d) $(x+7)$ and $(x+8)$

6. An equation, which remains unchanged when x is replaced by $\frac{1}{x}$ is called a/an: **(Board 2017)** 10301087
(a) Exponential equation
(b) Reciprocal equation
(c) Radical equation
(d) None of these

7. An equation of the type $3^x+3^{2-x}+6=0$ is a/an: **(Board 2014,16,22)** 10301088
(a) Exponential equation
(b) Reciprocal equation
(c) Radical equation
(d) None of these

8. The solution set of equation $4x^2-16=0$ is: **(Board 2014,15,20,24)** 10301089
(a) $\{\pm4\}$ (b) $\{4\}$
(c) $\{\pm2\}$ (d) ±2

9. An equation of the form $2x^4-3x^3+7x^2-3x+2=0$ is called a/an **(Board 2014)** 10301090
(a) Reciprocal equation **(Board 2018)**
(b) Radical equation
(c) Exponential equation
(d) None of these

10. The solution set of $25x^2-1=0$ is 10301091
(a) $\left\{\pm\frac{1}{5}\right\}$ (b) $\left\{-\frac{1}{5}\right\}$
(c) $\left\{+\frac{1}{5}\right\}$ (d) None of these

11. An equation of the form $2^{2x}-3.2^x+5=0$ is called a / an ___ equation. 10301092
(a) Exponential (b) Radical
(c) Reciprocal (d) None of these

12. The solution set of the equation $x^2-9=0$ is: **(Board 2022)** 10301093
(a) $\{\pm3\}$ (b) $\{3\}$
(c) $\{-3\}$ (d) $\{9\}$

13. An equation of type $x^4+x^3+x^2+x+1=0$ is called a/ an ……equation. 10301094
(a) Radical (b) Reciprocal
(c) Exponential (d) None of these

14. Solve the equation $5^{1+x} + 5^{1-x} = 26$
(a) $\{1\}$ (b) $\{\pm1\}$ 10301095
(c) $\{2\}$ (d) $\{\pm2\}$

15. The solution set of equation $2+9x=5x^2$ is: 10301096
(a) $\left\{\frac{-1}{5}, 2\right\}$ (b) $\left\{\frac{+1}{5}, 2\right\}$
(c) $\left\{\frac{1}{5}, -2\right\}$ (d) $\left\{\frac{-1}{5}, -2\right\}$

16. The solution set of equation $5x^2=30x$ is:
(Board 2024) 10301097
(a) $\{5, 30\}$ (b) $\{0,6\}$
(c) $\{0, -6\}$ (d) $\{5,0\}$

17. The solution set of equation $x^2-x-2=0$ is: 10301098
(a) $\{2, 1\}$ (b) $\{-2,1\}$
(c) $\{2, -1\}$ (d) $\{-2, -1\}$

18. The solution set of equation $x^2-16=0$ is: **(Board 2017)** 10301099
(a) $\{\pm4\}$ (b) $\{+4\}$
(c) $\{-4\}$ (d) ±4

19. The solution set of equation $x^2-7x+6=0$ is: 10301100
(a) $\{1, 6\}$ (b) $\{-1, -6\}$
(c) $\{-1, 6\}$ (d) $\{1, -6\}$

20. The solution set of equation $3x^2+4x=5$ is: 10301101
(a) $\left\{\frac{-2\pm\sqrt{19}}{3}\right\}$
(b) $\left\{\frac{2\pm\sqrt{19}}{3}\right\}$
(c) $\left\{\frac{4\pm\sqrt{19}}{3}\right\}$
(d) None of these

21. If b=0 in a quadratic equation $ax^2+bx+c=0$, then it is called: 10301102
(a) Pure quadratic equation
(b) Linear equation
(c) Quadratic equation
(d) Exponential equation

22. Sentences involving the sign…….. between two algebraic expressions are called equations. 10301103
(a) $<$ (b) $\geq$
(c) $=$ (d) $<$ or $>$

23. The standard form of the quadratic equation is $ax^2+bx+c=0$ where a, b, c are: 10301104
(a) Irrational numbers
(b) Rational numbers
(c) Real numbers
(d) Whole numbers

24. If a=0, in $ax^2+bx+c=0$, then it reduces to: 10301105
(a) pure quadratic equation
(b) linear equation
(c) quadratic equations
(d) exponential equation

25. How many linear factors a quadratic equation has? 10301106
(a) 1 (b) 2
(c) 3 (d) 4

26. What is the degree of quadratic equation? 10301107
(a) 1 (b) 2
(c) 3 (d) 4

27. The number of roots of a quadratic equation is: 10301108
(a) 1 (b) 2
(c) 3 (d) 4

28. Cancellation of x on both sides of $5x^2= 30x$ means: 10301109
(a) the loss of one root
(b) no loss of any root
(c) gain of one root
(d) undefined solution

29. What should be done to make the co-efficient of x^2 equal to 1, in $7x^2+2x-1=0$? 10301110
(a) multiply the equation by 7
(b) divide the equation by 7
(c) add 7 in both sides
(d) subtract 7 from both sides

30. What should be done to make the co-efficient of x^2 equal to 1 in $3x^2+7x=0$? **(Board 2021)** 10301111
(a) multiply the equation by $\frac{1}{3}$
(b) divide the equation by $\frac{1}{3}$
(c) add $\frac{1}{3}$ in both sides
(d) subtract $\frac{1}{3}$ from both sides

31. The value of variable of an equation not satisfying the equation is called: 10301112

(a) root (b) extraneous root
(c) exponent (d) solution set

32. The cancellation of x on both sides of the equation of the type $ax^2=bx$ means the loss of one root. That root is always equal to: 10301113

(a) 0 (b) 1
(c) a (d) b

33. If $y=x^{-1}$ and 3y=5, the value of x is:

(a) $\frac{5}{3}$ (b) $\frac{-5}{3}$ 10301114
(c) $\frac{-3}{5}$ (d) $\frac{3}{5}$

34. If $2^x=1$, then x = ... 10301115

(a) 0 (b) 1
(c) 2 (d) $\frac{1}{2}$

35. If $y=2^x$ and 8y =1, then, x =... 10301116

(a) 8 (b) $\frac{1}{8}$
(c) 3 (d) –3

35. Solution set of $5x^2 = 15x$ is: 10301116(a)

(a) {0} (b) {5} **(Board 2024)**
(c) {15} (d) {0,3}

ANSWERS KEY

1.	b	**2.**	c	**3.**	c	**4.**	a	**5.**	c
6.	b	**7.**	a	**8.**	c	**9.**	a	**10.**	a
11.	a	**12.**	a	**13.**	b	**14.**	b	**15.**	a
16.	b	**17.**	c	**18.**	a	**19.**	a	**20.**	a
21.	a	**22.**	c	**23.**	c	**24.**	b	**25.**	b
26.	b	**27.**	b	**28.**	a	**29.**	b	**30.**	a
31.	b	**32.**	a	**33.**	d	**34.**	a	**35.**	d
36.	d								

Q.2. Write short answers of the following questions. 10301117

(i) Solve $x^2 + 2x - 2 = 0$ **(Board 2017,21)**

Solution: $x^2 + 2x - 2 = 0$

$ax^2 + bx + c = 0$

$\Rightarrow$ a = 1 , b = 2 , c = –2

We know that

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-2\pm\sqrt{(2)^2-4(1)(-2)}}{2(1)}$$

$$x=\frac{-2\pm\sqrt{4+8}}{2}$$

$$x=\frac{-2\pm\sqrt{12}}{2}$$

$$x=\frac{-2\pm\sqrt{4\times3}}{2}$$

$$x=\frac{-2\pm2\sqrt{3}}{2}$$

$$x=\frac{\cancel{2}(-1\pm\sqrt{3})}{\cancel{2}}$$

$$x=-1\pm\sqrt{3}$$

$$S.S = \left\{-1\pm\sqrt{3}\right\}$$

(ii) Solve by factorization $5x^2 = 15x$ 10301118

Solution: **(Board 2014,15,16,19,20)**

$5x^2 = 15x$

$5x^2 - 15x = 0$

$5x(x-3) = 0$

$5x(x-3) = 0$

$5x = 0$ or $x - 3 = 0$

$x = \frac{0}{5}$ or $x = 3$

$x = 0$ or $x = 3$

Solution set is { 0, 3}

(iii) Write in standard form $\frac{1}{x+4}+\frac{1}{x-4}=3$

Solution: $\frac{1}{x+4}+\frac{1}{x-4}=3$ 10301119 **(Board 2014,16,24)**

$$\frac{(x-4)+(x+4)}{(x+4)(x-4)}=3$$

$$\frac{x-\cancel{4}+x+\cancel{4}}{(x)^2-(4)^2}=3$$

$$\frac{2x}{x^2-16}=3$$

$2x = 3(x^2 - 16)$

$2x = 3x^2 - 48$

$0 = 3x^2 - 2x - 48$

$3x^2 - 2x - 48 = 0$

(iv) Write the name of methods for solving a quadratic equation. 10301120

Ans. Following three methods are used for solving the quadratic equation.

(i) Factorization method
(ii) Completing square method
(iii) Quadratic formula method

(v) Solve $\left(2x-\frac{1}{2}\right)^2=\frac{9}{4}$ 10301121

Solution: $\left(2x-\frac{1}{2}\right)^2=\frac{9}{4}$ **(Board 2014,20,21,23)**

Taking square root

$$\sqrt{\left(2x-\frac{1}{2}\right)^{\cancel{2}}}=\pm\sqrt{\frac{9}{4}}$$

$$2x-\frac{1}{2}=\pm\frac{3}{2}$$

$$2x=\pm\frac{3}{2}+\frac{1}{2}$$

$$2x=\frac{\pm3+1}{2}$$

$$x=\frac{\pm3+1}{2\times2}$$

$$x=\frac{\pm3+1}{4}$$

$\Rightarrow$ $x=\frac{3+1}{4}$ or $x=\frac{-3+1}{4}$

$x=\frac{4}{4}$ or $x=\frac{-2}{4}$

$x=1$ or $x=\frac{-1}{2}$

The Solution set is $\left\{1,\frac{-1}{2}\right\}$

(vi) Solve $\sqrt{3x+18}=x$ 10301122

Solution: **(Board 2014,15)**

$\sqrt{3x+18}=x$ .....(i)

Taking square of both sides

$(\sqrt{3x+18})^2=(x)^2$

$3x+18=x^2$

$\Rightarrow$ $x^2-3x-18=0$

$x^2-6x+3x-18=0$

$x(x-6)+3(x-6)=0$

$(x-6)(x+3)=0$

$x-6=0$ or $x+3=0$

$x=6$ or $x=-3$

Checking

Put $x=6$ in equation (i)	Put $x=-3$ in equation (i)
$\sqrt{3x+18}=x$	$\sqrt{3x+18}=x$
$\sqrt{3(6)+18}=6$	$\sqrt{3(-3)+18}=-3$
$\sqrt{18+18}=6$	$\sqrt{-9+18}=-3$
$\sqrt{36}=6$	$\sqrt{9}=-3$
$6=6$	$3=-3$
which is true	which is not true

As -3 is an extraneous root

So, the solution set is $\{6\}$

(vii) Define quadratic equation 10301123

Quadratic Equation **(Board 2014,15,22,23)**

An equation which contains the square of the unknown (variable) quantity, but no higher power, is called a quadratic equation. Standard form of quadratic equation in one variable x is $ax^2+bx+c=0$, where $a, b, c \in R$ and $a \neq 0$.

(viii) Define reciprocal equation 10301124

Reciprocal Equation **(Board 2014,15,19,21)**

An equation is said to be a reciprocal equation, if it remains unchanged, when x is replaced by $\frac{1}{x}$.

Examples

(i) $ax^4+bx^3+cx^2+bx+a=0$ OR

(ii) $a\left(x^2+\frac{1}{x^2}\right)+b\left(x+\frac{1}{x}\right)+c=0$

(ix) Define exponential equation: 10301125

Exponential Equation: **(Board 2015,20,21)**

An equation in which variable occurs in exponent is called exponential equation.

Example: $2^x-8=0$

(x) Define radical equation 10301126

Radical Equation **(Board 2014,17,18,22,23,24)**

An equation involving expression of variable under the radical sign is called radical equation.

Examples: **(i)** $\sqrt{x+3}=1$

(ii) $\sqrt{x-1}=\sqrt{x-2}$

(xi) Define Extraneous root. 10301127

Extraneous root:

A root of an equation, which do not satisfy the original equation is called Extraneous root.

(xii) Write two pure quadratic equations. 10301128

Two pure quadratic equations are given as:

(i) $x^2 - 16 = 0$

(ii) $4x^2 = 7$

(xii) Solve by factorization $x^2 - x - 2 = 0$

Solution: 10301129

$$x^2 - x - 2 = 0$$
$$x^2 - 2x + x - 2 = 0$$
$$x(x-2) + 1(x-2) = 0$$
$$(x-2)(x+1) = 0$$
$$x - 2 = 0 \quad \text{or} \quad x + 1 = 0$$
$$x = 2 \quad \text{or} \quad x = -1$$
$$S.S = \{2, -1\}.$$

(xiii) Using quadratic formula write the solution set of $x^2 + x - 2 = 0$ 10301130

Solution:

$$x^2 + x - 2 = 0$$
$$ax^2 + bx + c = 0$$
$$\Rightarrow \quad a = 1,\ b = 1,\ c = -2$$

We know that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-2)}}{2(1)}$$
$$x = \frac{-1 \pm \sqrt{1+8}}{2}$$
$$x = \frac{-1 \pm \sqrt{9}}{2}$$
$$x = \frac{-1 \pm 3}{2}$$

$x = \frac{-1+3}{2}$	$x = \frac{-1-3}{2}$
$x = \frac{2}{2}$	$x = \frac{-4}{2}$
$x = 1$	$x = -2$

So,

$$S.S = \{1, -2\}$$

(xiv) Find solution set of $25x^2 - 1 = 0$ by factorization. 10301131

Solution:

$$25x^2 - 1 = 0$$
$$(5x)^2 - (1)^2 = 0$$
$$(5x+1)(5x-1) = 0$$
$$5x + 1 = 0 \quad \text{or} \quad 5x - 1 = 0$$
$$5x = -1 \quad \text{or} \quad 5x = 1$$
$$x = \frac{-1}{5} \quad \text{or} \quad x = \frac{1}{5}$$

Solution set is $\left\{\frac{-1}{5}, \frac{1}{5}\right\}$

(xv) Find solution set of $x^2 - 9 = 0$ by factorization. **(Board 2015)** 10301131

Solution:

$$x^2 - 9 = 0$$
$$(x)^2 - (3)^2 = 0$$
$$(x+3)(x-3) = 0$$
$$x + 3 = 0 \quad \text{or} \quad x - 3 = 0$$
$$x = -3 \quad \text{or} \quad x = 3$$

So, $S.S = \{-3, 3\}$

(xvi) Solve by factorization $5x^2 = 15$

(Board 2105, 16) 10301132

Solution:

$$5x^2 = 15$$
$$5x^2 - 15 = 0$$
$$5(x^2 - 3) = 0$$
$$\because \quad 5 \neq 0$$

So,

$$x^2 - 3 = 0$$
$$(x^2) - (\sqrt{3})^2 = 0$$
$$(x + \sqrt{3})(x - \sqrt{3}) = 0$$
$$x + \sqrt{3} = 0 \quad \text{or} \quad x - \sqrt{3} = 0$$
$$x = -\sqrt{3} \quad \text{or} \quad x = \sqrt{3}$$

So,

Solution set is $\{\sqrt{3}, -\sqrt{3}\}$

(xvii) Write standard quadratic equation and also write quadratic formula to solve it. **(Board 2018)** 10301133

Ans. The standard form of a quadratic equation in one variable x is $ax^2+bx+c=0$, where $a,b,c \in R$ and $a \neq 0$.

The quadratic formula to solve a quadratic equation is $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$.

Q.3 Fill in the blanks:

i. The standard form of the quadratic equation is_____. 10301134

ii. The number of methods to solve a quadratic equation are…... 10301135

iii. The name of the method to derive a quadratic formula is …... 10301136

iv. The solution of the equation $ax^2+bx+c=0$, $a \neq 0$_____. 10301137

v. The solution set of $25x^2-1=0$ is_____. 10301138

vi. An equation of the form $2^{2x}-3.2^x+5=0$ is called a/an _____equation. 10301139

vii. The solution set of the equation $x^2-9=0$ is_____. 10301140

viii. An equation of the type $x^4+x^3+x^2+x+1=0$ is called an____ equation. 10301141

ix. A root of an equation, which do not satisfy the equation is called _____root. 10301142

x. An equation involving expression of the variable under_____ is called radical equation. 10301143

Answers:

i. $ax^2+bx+c=0, \quad a \neq 0$

ii. Three

iii. Completing square

iv. $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

v. $\left\{\pm\dfrac{1}{5}\right\}$

vi. Exponential

vii. $\{\pm 3\}$

viii. Reciprocal

ix. Extraneous

x. Radical Sign