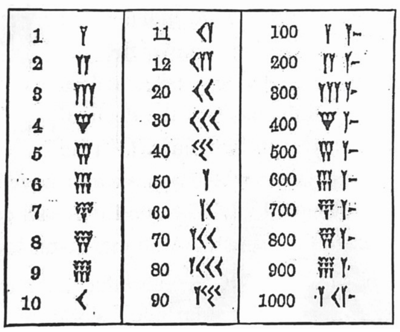
UNIT-1

Real Numbers

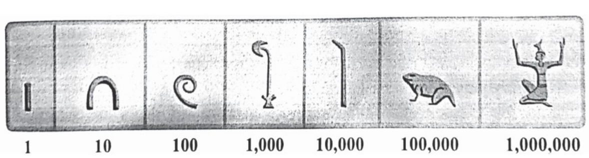
Introduction to Real Numbers

The history of numbers comprises thousands of years, from ancient civilization to the modern Arabic system.

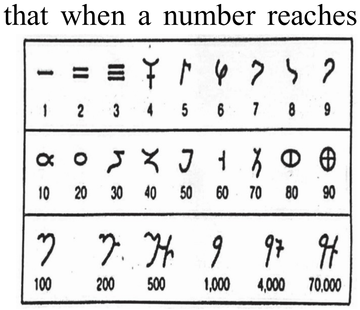
Sumerians: (4500–1900BCE) used a sexagesimal (base 60) system for counting. The Sumerians used a small cone, bead, large cone, large perforated cone, sphere and perforated sphere, corresponding to 1, 10, 60 (a large unit), 600.



Egyptians: (3000–2000 BCE) used a decimal (base 10) system for counting. Here are some of the symbols used by the Egyptians, as shown in the figure below: The Egyptians usually wrote numbers left to right, starting with the highest denominator. For example, 2525 would be written with 2000 first, then 500, 20, and 5.



Romans: (500BCE-500CE) used the Roman materials system for counting. Roman numerals represent a number system that was widely used throughout Europe as the standard writing system until the late Middle ages. The ancient Romans explained that when a number reaches 10 it is not easy to count on one's fingers. Therefore, there was a need to create a proper number system that could be used for trade and communications. Roman numerals use 7 letters to represent different numbers. These are I, V X, L,C,D and M which represent the numbers 1,5,10,50,100,500 and 1000 respectively.



Indians: (500-1200)CE developed the concept of zero (0) and made a significant contribution to the decimal (base 10) system.

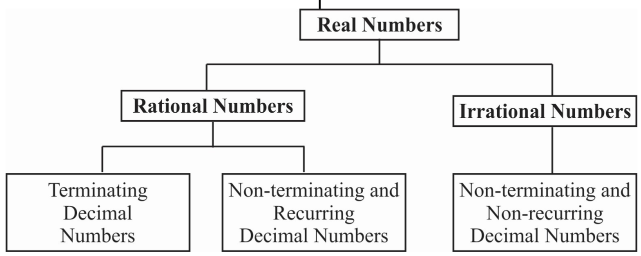
Ancient Indian mathematicians have contributed immensely to the field of mathematics. The invention of zero is attributed to Indians, and this contribution outweighs all others made by any other nation since it is the basis of the decimal number system, without which no advancement in mathematics would have been possible. The number system used today was invented by Indians, and it is still called Indo-Arabic numerals because Indians invented them and the Arab merchants took them to the Western world.

Arabs: (800-1500CE) introduced Arabic numerals (0-9) to Europe. The Islamic world underwent significant developments in mathematics. Muhammad bin Musa al-Khwarzimi played a key role in this transformation, introducing algebra as a distinct field in the 9th century. Al-Khwarzimi's approach departing from earlier arithmetical traditions, laid the groundwork for a arithmetization of algebra, influencing mathematical thought for an extended period. Successors like Al-Karaji expanded on his work, contributing to advancements in various mathematical domains. The practicality and broad applicability of these mathematical methods facilitated the dissemination of Arabic mathematics to the West, contributing substantially to the evolution of Western mathematics.

Modern era (1700-present): Developed modern number systems e.g., binary system base -2) and hexadecimal system (base-16).

The Arabic system is the basis for modern decimal system used globally today. Its development and refinement comprise thousands of years from ancient Sumerians to modern mathematicians.

In the modern era, the set [1,2,3,…] was adopted as the counting set. This counting set represents the set of natural numbers was extended to set of real numbers which is used most frequently in everyday life.



Rational Numbers:

The set of rational numbers is defined as the set of numbers that contains those elements which can be expressed as quotient of two integers. For example, $\frac{2}{3}, \frac{4}{5}, \frac{6}{7}$....*etc.*

$$Q = \left\{ \frac{p}{q}; p, q \in Z \wedge q \neq 0 \right\}$$

Irrational Numbers:

The set of irrational numbers Q' contains those elements which can not be expressed as quotient of integers.

$$Q' = \left\{ x \neq \frac{p}{q}; p, q \in Z \wedge q \neq 0 \right\}$$

For example, $\pi, e, \sqrt{2}, \sqrt{3}, \sqrt{5}$ and $\sqrt{7}$ *etc.* are irrational numbers.

Real Numbers:

The set of Real numbers is the union (combination) of the set of rational numbers and irrational numbers i,e., $R = Q \cup Q'$

Decimal Representation of Rational Numbers

(i) Terminating Decimal Numbers:

A decimal number with a finite number of digits after the decimal point is called a terminating decimal number.

For example $\frac{1}{4} = 0.25$, $\frac{8}{25} = 0.32$, $\frac{3}{8} = 0.375$, $\frac{4}{5} = 0.8$ are all terminating decimals.

(ii) Recurring and Non-Terminating Decimal Numbers

The decimal numbers with a repeating pattern of digits after the decimal point are called recurring decimal numbers.

For example,

$\frac{1}{3} = 0.333... = 0.\overline{3}$ (the 3 repeats infinitely)

$\frac{1}{6} = 0.1666... = 0.1\overline{6}$ (the 6 repeats infinitely)

Example 1: Identify the following decimal numbers as rational or irrational numbers:

(i) 0.35 **09301001** (ii) 0.444… **09301002**

(iii) $3.\overline{5}$ **09301003**

(iv) 3.36788542… **09301004**

(v) 1.709975947… **09301005**

Solution:

(i) 0.35 is a terminating decimal number, therefore it is a rational number.

(ii) 0.444… is a recurring decimal number, therefore it is a rational number.

(iii) $3.\overline{5}$ = 3.5555… is a recurring decimal number, therefore it is a rational number.

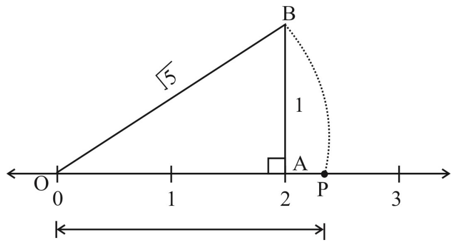
(iv) 3.3678542… is a non-terminating and non-recurring decimal number. Therefore, it represents an irrational number.

(v) 1.709975947… is a non-terminating and non-recurring decimal number, it is an irrational number.

Example 2: Represent $\sqrt{5}$ on a number line.

Solution: **09301006**

$\sqrt{5}$ can be located on the real line by geometric construction. As $\sqrt{5} = 2.236...$ which is near to 2. Mark a line of $m\overline{AB} = 1$ unit at A, where $m\overline{OA} = 2$ units, and we have a right-angle triangle *OAB*. By using Pythagoras theorem



$$\left(m\overline{OB}\right)^2 = \left(m\overline{OA}\right)^2 + \left(m\overline{AB}\right)^2$$

$$\Rightarrow m\overline{OB} = \sqrt{(2)^2 + (1)^2} = \sqrt{4+1} = \sqrt{5}$$

$$\Rightarrow m\overline{OB} = \sqrt{5}$$

Draw an arc of radius $m\,\overline{OB} = \sqrt{5}$ taking O as centre, we got point "*P*" representing $\sqrt{5}$ on the number line

So, $\left|\overline{OP}\right| = \sqrt{5}$

Example 3: Express the following recurring decimals as the rational number $\frac{p}{q}$, where p and q are integers.

(i) $0.\overline{5}$ **09301007** (ii) $0.\overline{93}$ **09301008**

(i) **$0.\overline{5}$**

Solution

$0.\overline{5}$

Let $x = 0.\overline{5}$

$x = 0.5555\ldots$ ________(i)

Multiply both sides by 10

$10x = 10(0.5555\ldots)$

$10x = 5.555\ldots$_________(ii)

Subtracting eq. (i) from eq. (ii)

$10x - x = (5.555\ldots) - (0.5555\ldots)$

$9x = 5$

$\Rightarrow \quad x = \frac{5}{9}$

Which shows the rational number in the form $\frac{p}{q}$.

(ii) Let $x = 0.\overline{93}$

Solution

$0.\overline{93}$

$x = 0.93939393\ldots$ ____________(i)

Multiply both sides by 100, we get

$100x = 100\ (0.93939393\ldots)$

$100x = 93.939393\ldots \quad \ldots$(ii)

Subtracting (i) from (ii)

$100x - x = 93.939393\ldots - 0.93939393\ldots$

$99x = 93$

$x = \frac{93}{99}$ which is a rational number.

Example 4: Insert two rational numbers between 2 and 3. **09301009**

Solution

There are infinite rational numbers between 2 and 3.

We find any two of them

For this, find the average of 2 and 3 = $\frac{2+3}{2} = \frac{5}{2}$

So, $\frac{5}{2}$ is a rational number between 2 and 3, to find another rational number between 2 and 3 we will again find average of $\frac{5}{2}$ and 3 i.e,

$$= \left(\frac{5}{2} + 3\right) \div 2$$

$$= \left(\frac{5+6}{2}\right) \div \frac{2}{1}$$

$$= \left(\frac{11}{2}\right) \times \frac{1}{2} = \frac{11}{4}$$

Hence two rational numbers between 2 and 3 are $\frac{5}{2}$ and $\frac{11}{4}$.

> **Try Yourself**
> What will be the product of two irrational numbers?

Properties of Real Numbers

All calculations involving addition, subtraction, multiplication, and division of real numbers are based on their properties. In this section, we shall discuss these properties.

Additive properties

Name of the property	$\forall a$, b, c $\in$ R	Examples
Closure	$a + b \in R$	$2 + 3 = 5 \in R$
Commutative	$a + b = b + a$	$2 + 5 = 5 + 2$ $7 = 7$

		2 + 8 = 5 + 5 10 = 10
Identity	$a+0=a=0+a$	5 + 0 = 5 = 0 + 5
Inverse	$a+(-a)=-a+a=0$	6 + (–6) = (–6) + 6 = 0

Multiplicative properties

Name of the property	$\forall a$, b, c ∈ R	Examples
Closure	$ab \in R$	$2 \times 5 = 10 \in R$
Commutative	$ab = ba$	2 × 3 = 3 × 2 = 6
Associative	$a(bc) = (ab)c$	2 × (3 × 5) = (2 × 3) × 5 2 × 15 = 6 × 5 30 = 30
Identity	$a \times 1 = 1 \times a = a$	5 × 1 = 1 × 5 = 5
Inverse	$a \times \frac{1}{a} = \frac{1}{a} \times a = 1$	$7 \times \frac{1}{7} = \frac{1}{7} \times 7 = 1$

Do you know?

0 and 1 are the additive and multiplicative identities of real numbers respectively.

Remember!

$0 \in R$ has no multiplicative Inverse.

Distributive Properties

For all real numbers *a, b, c*

(i) $a(b+c) = ab + ac$ is called left distributive property of multiplication over addition.

(ii) a(b–c) = ab–ac is called left distributive property of multiplication over subtraction.

(iii) $(a+b)c = ac + bc$ is called right distributive property of multiplication over addition.

(iv) (a–b)c = ac–bc is called right distributive property of multiplication over subtraction.

Properties of Equality of Real number

i.	Reflexive property	$\forall a \in R,\ a = a$
ii.	Symmetric property	$\forall a,b \in R,\ a = b \Rightarrow b = a$
iii.	Transitive property	$\forall a,b,c \in R,\ a = b \wedge b = c \Rightarrow a = c$
iv	Additive properly.	$\forall a,b,c \in R,\ a = b \Rightarrow a + c = b + c$
v.	Multiplicative property	$\forall a,b,c \in R,\ a = b \Rightarrow ac = bc$
vi.	Cancellation property w.r.t addition	$\forall a,b,c \in R,\ a + c = b + c \Rightarrow a = b$
vii.	Cancellation property w.r.t multiplication	$\forall a$,b,c ∈ R, and c ≠ 0 ac = bc ⇒ a = b

Order Properties

i.	Trichotomy property	$\forall a,b \in R$, either $a = b$ or $a > b$ or $a < b0$
ii.	Transitive Property	$\forall a,b,c \in R$ • $a > b \wedge b > c \Rightarrow a > c$ • $a < b \wedge b < c \Rightarrow a < c$
iii.	Additive property	$\forall a,b,c \in R$

iv	Multiplicative property	$\forall a,b,c \in R$ • $a > b \Rightarrow ac > bc$ if $c > 0$ • $a < b \Rightarrow ac < bc$ if $c > 0$ • $a > b \Rightarrow ac < bc$ if $c < 0$ • $a < b \Rightarrow ac > bc$ if $c < 0$ • $a > b \wedge c > d \Rightarrow ac > bd$ • $a < b \wedge c < d \Rightarrow ac < bd$
v	Reciprocal property	• $\forall a,b \in R$ • $a < \mathrm{b} \Rightarrow \frac{a}{\mathrm{c}} < \frac{\mathrm{b}}{\mathrm{c}}$ if c > 0 • $a < \mathrm{b} \Rightarrow \frac{a}{\mathrm{c}} > \frac{\mathrm{b}}{\mathrm{c}}$ if c < 0 • $a > b \Rightarrow \frac{a}{\mathrm{c}} > \frac{\mathrm{b}}{\mathrm{c}}$ if c > 0 • $a > b \Rightarrow \frac{a}{\mathrm{c}} < \frac{\mathrm{b}}{\mathrm{c}}$ if c > 0 • $a > b \Rightarrow \frac{1}{a} < \frac{1}{b}$
vi	Reciprocal property	$\forall$ a, b$\in$ R and have same sign • $a < \mathrm{b} \Rightarrow \frac{1}{a} > \frac{1}{\mathrm{b}}$ • $a < \mathrm{b} \Rightarrow \frac{1}{a} < \frac{1}{\mathrm{b}}$

Example 5: If $a = \frac{2}{3}, b = \frac{3}{2}, c = \frac{5}{3}$ then verify the distributive property over addition. **09301010**

Solution: (i) Left distributive property

$a(b+c) = ab + ac$

L.H.S = $a(b+c)$	R.H.S = $ab + ac$
$= \frac{2}{3}\left(\frac{3}{2}+\frac{5}{3}\right)$	$= \left(\frac{2}{3}\right)\left(\frac{3}{2}\right)+\left(\frac{2}{3}\right)\left(\frac{5}{3}\right)$
$= \frac{2}{3}\left(\frac{9+10}{6}\right)$	$= 1 + \frac{10}{9}$
$= \frac{2}{3}\left(\frac{19}{6}\right)$	$= \frac{9+10}{9}$
$= \frac{19}{9}$	$= \frac{19}{9}$

L.H.S = R.H.S

Hence proved.

(ii) Right distributive property

$(a+b)c = ac + bc$

L.H.S = $(a+b)c$	R.H.S = $ac + bc$
$= \left(\frac{2}{3}+\frac{3}{2}\right)\frac{5}{3}$	$= \left(\frac{2}{3}\right)\left(\frac{5}{3}\right)+\left(\frac{3}{2}\right)\left(\frac{5}{3}\right)$
$= \left(\frac{4+9}{6}\right)\frac{5}{3}$	$= \frac{10}{9}+\frac{15}{6}$
$= \left(\frac{13}{6}\right)\left(\frac{5}{3}\right)$	$= \frac{20+45}{18}$
$= \frac{65}{18}$	$= \frac{65}{18}$

L.H.S = R.H.S

Hence Proved

Example 6: Identify the property that justifies the statement

(i) If a >13 then a + 2 > 15 **09301011**

(ii) If 3 < 9 and 6 < 12 then 9 < 21 **09301012**

(iii)If 7 > 4 and 5 > 3 then 35 > 12 **09301013**

(iv)If –5 < –4 $\Rightarrow$ 20 > 16 **09301014**

Solution

(i) $a > 13$

Add 2 on both sides

$a + 2 > 13 + 2$

$a + 2 > 15$ (order property w.r.t addition)

(ii) As $3 < 9$ and $6 < 12$

$\Rightarrow 3 + 6 < 9 + 12$

$9 < 21$ (order property w.r.t addition)

(iii) $7 > 4$ and $5 > 3$

$\Rightarrow 7 \times 5 > 4 \times 3$

$\Rightarrow 35 > 12$

(order property w.r.t multiplication)

(iv) As $-5 < -4$

$-5 \times -4 > -4 \times -4$

$\Rightarrow 20 > 16$ (order property w.r.t multiplication)

Exercise 1.1

Q.1 Identify each of the following as a rational or irrational numbers:

Solutions

Rational numbers

(i) 2.353535 **09301015** (ii) $0.\overline{6}$ **09301016**

(ix) $\frac{15}{4}$ **09301017** (x) $\left(2-\sqrt{2}\right)\left(2+\sqrt{2}\right)$ **09301018**

Irrational numbers

(iii) 2.236067… **09301019** (iv) $\sqrt{7}$ **09301020**

(v) e **09301021** (vi) π **09301022**

(vii) $5+\sqrt{11}$ **09301023** (viii) $\sqrt{3}+\sqrt{13}$ **09301024**

Q.2 Represent the following numbers on number line:

(i) $\sqrt{2}$ **09301025**

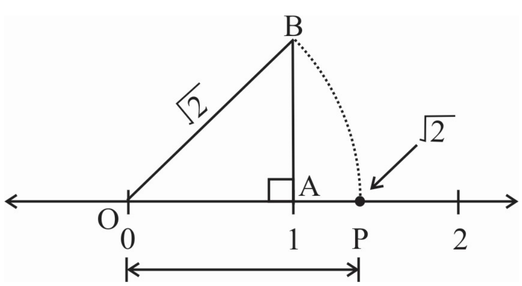
Solution

$\sqrt{2}$ can be located on the real line by geometric construction. Mark a line of $m\overline{AB} = 1$ unit at A, where $m\overline{OA} = 1$ unit, and we have a right-angle triangle *OAB*. By using Pythagoras theorem

$$\left(m\overline{OB}\right)^2 = \left(m\overline{OA}\right)^2 + \left(m\overline{AB}\right)^2$$

$$\sqrt{\left(m\overline{OB}\right)^2} = \sqrt{\left(m\overline{OA}\right)^2 + \left(m\overline{AB}\right)^2}$$

$$m\overline{OB} = \sqrt{(1)^2 + (1)^2} = \sqrt{1+1} = \sqrt{2}$$



Taking O as centre, draw an arc of radius $m\,\overline{OB} = \sqrt{2}$ Which cut the number line at P. W e get point "*P*" representing $\sqrt{2}$ on the number line

So, $\left|\overline{OP}\right| = \sqrt{2}$

(ii) $\sqrt{3}$ **09301026**

Solution

$\sqrt{3}$ can be located on the real line by geometric method. Mark a line of $m\overline{AB} = 1$ unit at A, With centre at A draw an arc of radius 2 units above the line. From point B draw the line segment so that it cuts the arc at C. Join A to C. We have a right-angled ΔABC *in which* $m\overline{AB} = 1$ unit and $m\overline{AC} = 2$ units. By using Pythagoras theorem.

$$\left(m\overline{AC}\right)^2 = \left(m\overline{AB}\right)^2 + \left(m\overline{BC}\right)^2$$

$$(2)^2 = (1)^2 + \left(m\overline{BC}\right)^2$$

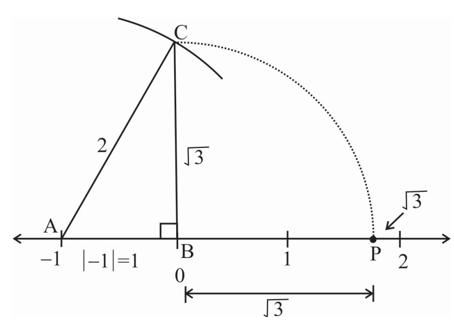
$$4 = 1 + \left(m\overline{BC}\right)^2$$

$$4 - 1 = \left(m\overline{BC}\right)^2$$

$$3 = \left(m\overline{BC}\right)^2$$

$$\sqrt{\left(m\overline{BC}\right)^2} = \sqrt{3}$$

$$m\overline{BC} = \sqrt{3}$$

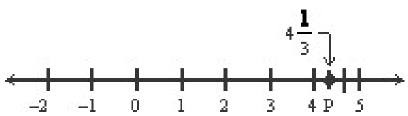


Now consider B is at 0. Taking B as centre, draw an arc of radius $m\,\overline{BC} = \sqrt{3}$, which cut the number line at P. We get point "*P*" representing $\sqrt{3}$ on the number line

So, $\left|\overline{BP}\right| = \sqrt{3}$

(iii) $4\frac{1}{2}$ **09301027**

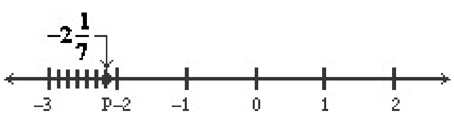
Solution



Point P represents $4\frac{1}{3}$ on the number line.

(iv) $2\frac{1}{7}$ **09301028**

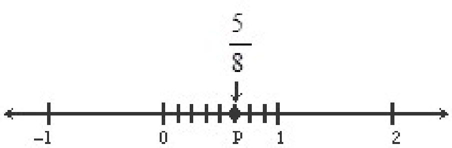
Solution



Point P represents $-2\frac{1}{7}$ on the number line.

(v) $\frac{5}{8}$ **09301029**

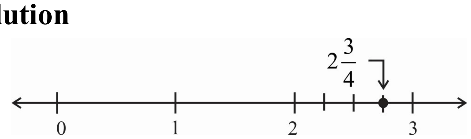
Solution



Point P represents $\frac{5}{8}$ on the number line.

(vi) $2\frac{3}{4}$ **09301030**

Solution



Point P represents $2\frac{3}{4}$ on the number line.

Q.3 Express the following as a rational number $\frac{p}{q}$ where p and q are integers and $q \neq 0$.

(i) $x = 0.\bar{4}$ **09301031**

Solution

Let $x = 0.\bar{4}$

$x = 0.4444\ldots$________ (i)

Multiplying both sides by "10" , we get

$10x = 10\,(0.4444\ldots)$

$10x = 4.444\ldots$________ (ii)

Subtracting eq.(i) from (ii)

$10x - x = (4.444\ldots) - (0.4444\ldots)$

$9x = 4$

$x = \frac{4}{9}$

$\Rightarrow\ 0.\bar{4} = \frac{4}{9}$

(ii) $0.\overline{37}$ **09301032**

Solution

Let $x = 0.\overline{37}$

$x = 0.37373737\ldots$________ (i)

Multiplying both sides by "100"

$100x = 100\ (0.37373737\ldots)$

$100x = 37.373737\ldots$________ (ii)

Subtracting eq.(i) from (ii)

$100x - x = (37.373737\ldots) - (0.37373737\ldots)$

$99x = 37$

$x = \frac{37}{99}$

$\Rightarrow\ 0.\overline{37} = \frac{37}{99}$

(iii) $0.\overline{21}$ **09301033**

Solution

Let $x = 0.\overline{21}$

$x = 0.21212121\ldots$ _________(i)

Multiplying both sides by "100"

$100x = 100\ (0.21212121\ldots)$

$100x = 21.212121\ldots$ _______(ii)

Subtracting eq.(i) from (ii)

$100x - x = (21.212121\ldots) - (0..21212121\ldots)$

$99x = 21$

$x = \frac{21}{99} = \frac{7}{33}$

$\Rightarrow\ 0.\overline{21} = \frac{7}{33}$

Q.4 Name the property used in the following.
Solution
(i) 09301034

Sr. No.		Property Name
(i)	**(a + 4) + b = a + (4+b)**	Associative property w.r.t addition
(ii)	$\sqrt{2}+\sqrt{3}=\sqrt{3}+\sqrt{2}$	Commutative property w.r.t addition
(iii)	$x - x = 0$	Additive Inverse
(iv)	**a(b+c) = a b + a c**	Left distributive property of multiplication over addition.
(v)	**16 + 0 = 16**	Additive Identity
(vi)	**100×1 = 100**	Multiplicative identify
(vii)	**4 × (5×8) = (4×5) × 8**	Associative property w.r.t multiplication
(viii)	**ab = ba**	Commutative property w.r.t multiplication.

5. Name the property used in the following:
Solutions
(i) $-3 < -1 \Rightarrow 0 < 2$ 09301035
Additive property of inequality
(ii) If a < b then $\frac{1}{a} > \frac{1}{b}$ 09301036
Reciprocal property
(iii) If a < b then a+c < b+c 09301037
Additive property of inequality
(iv) If ac < bc and c > 0 then a < b
Cancellation property of inequality w.r.t multiplication.
(v) If ac < bc and c < 0 then a > b 09301038
Cancellation property of inequality w.r.t multiplication.
(vi) Either a > b or a = b or a < b
Trichotomy property
6. Insert two rational numbers between
(i) $\frac{1}{3}$ and $\frac{1}{4}$ 09301039
Solution
Two rational numbers between $\frac{1}{3}$ and $\frac{1}{4}$

Average of $\frac{1}{3}$ and $\frac{1}{4} = \left(\frac{1}{3}+\frac{1}{4}\right) \div 2$

$= \left[\frac{4+3}{12}\right]\times\frac{1}{2}$

$= \frac{7}{12}\times\frac{1}{2}=\frac{7}{24}$

Now we find average of $\frac{1}{3}$ and $\frac{7}{24}$

$= \left(\frac{1}{3}+\frac{7}{24}\right)\div 2$

Average of $\frac{1}{3}$ and $\frac{7}{24}$

$= \frac{8+7}{24}\times\frac{1}{2}$

$= \frac{15}{24}\times\frac{1}{2}=\frac{15}{48}=\frac{5}{16}$

Thus $\frac{5}{16}$ and $\frac{7}{24}$ are two rational numbers between $\frac{1}{3}$ and $\frac{1}{4}$.

(ii) 3 and 4 09301040
Solution
Two rational numbers between 3 and 7.

Average of 3 and 4 = $\frac{3+4}{2}=\frac{7}{2}$

Average of $\frac{7}{2}$ and 4 $= \left(\frac{7}{2}+4\right)\div 2$

$= \left(\frac{7+8}{2}\right)\div\frac{2}{1}$

$= \frac{15}{2}\times\frac{1}{2}$

$= \frac{15}{4}$

Thus $\frac{7}{2}$ and $\frac{15}{4}$ are two rational numbers between 3 and 4.

(iii) $\frac{3}{5}$ and $\frac{4}{5}$ 09301041
Solution
Two rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$

Average of $\frac{3}{5}$ and $\frac{4}{5} = \left(\frac{3}{5}+\frac{4}{5}\right)\div 2$

$$= \left(\frac{3+4}{5}\right) \times \frac{1}{2}$$

$$= \frac{7}{5} \times \frac{1}{2} = \frac{7}{10}$$

Average of $\frac{7}{10}$ and $\frac{4}{5} = \left(\frac{7}{10} + \frac{4}{5}\right) \div 2$

$$= \left(\frac{7+8}{10}\right) \times \frac{1}{2}$$

$$= \frac{15}{10} \times \frac{1}{2}$$

$$= \frac{15}{20} = \frac{3}{4}$$

Thus $\frac{7}{10}$ and $\frac{3}{4}$ are two rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Radical Expressions

If n is a positive integer greater than 1, and a is a real number, then any real number x such that $x = \sqrt[n]{a}$ is called n^{th} root of a.

Here $\sqrt{\ }$ is called radical , and n is the index of radical. A real number under the radical sign is called a radicand. $\sqrt[3]{5}, \sqrt[5]{7}$ are examples of radical form.

Laws of Radicals and Indices

Laws of Radical	Laws of Indices
(i) $\sqrt[n]{ab} = \sqrt[n]{a}.\sqrt[n]{b}$	(i) $a^m.a^n = a^{m+n}$
(ii) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$	(ii) $(a^m)^n = a^{mn}$
(iii) $\sqrt[n]{a^m} = (\sqrt[n]{a})^m$	(iii) $(ab)^n = a^n b^n$
(iv) $\left(\sqrt[n]{a}\right)^n = (a^{\frac{1}{n}})^n = a$	(iv) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
	(v) $\frac{a^m}{a^n} = a^{m-n}$
	(vi) $a^0 = 1$

Example 7

Simplify the following:

(i) $\sqrt[4]{16x^4y^8}$ **09301042**

Solution

$\sqrt[4]{16x^4y^8}$

$= (16x^4y^8)^{\frac{1}{4}}$ $\quad \sqrt[n]{a} = a^{\frac{1}{n}})$

$= (16)^{\frac{1}{4}}(x^4)^{\frac{1}{4}}(y^8)^{\frac{1}{4}}$ $\quad \because (ab)^m = a^m b^m$

$= 2^{4\cdot\frac{1}{4}} \times x^{4\cdot\frac{1}{4}} \times y^{8\cdot\frac{1}{4}}$ $\quad \because (a^m)^n = a^{mn}$

$= 2xy^2$

(ii) $\sqrt[3]{\mathbf{27x^6y^9z^3}}$ **09301043**

Solution

$\sqrt[3]{27x^6y^9z^3}$

$= \left(27x^6y^9z^3\right)^{\frac{1}{3}}$ $\quad \because \sqrt[n]{a} = a^{\frac{1}{n}}$

$= \left(27\right)^{\frac{1}{3}}\left(x^6\right)^{\frac{1}{3}}\left(y^9\right)^{\frac{1}{3}}\left(z^3\right)^{\frac{1}{3}}$ $\quad \because \left(ab\right)^m = a^m b^m$

$= \left(3^3\right)^{\frac{1}{3}}\left(x^6\right)^{\frac{1}{3}}\left(y^9\right)^{\frac{1}{3}}\left(z^3\right)^{\frac{1}{3}}$ $\quad \because (ab)^m = a^m b^m$

$= 3^{3\times\frac{1}{3}} \cdot x^{6\times\frac{1}{3}} \cdot y^{9\times\frac{1}{3}}.z^{3\times\frac{1}{3}}$

$= 3x^2y^3z$

(iii) $\mathbf{(64)^{-\frac{4}{3}}}$ **09301044**

Solution

$(64)^{-\frac{4}{3}}$

$= \frac{1}{(64)^{\frac{4}{3}}}$

$= \frac{1}{(4)^{\frac{4}{3}}} = \frac{1}{4^{3\times\frac{4}{3}}}$

$= \frac{1}{4^4} = \frac{1}{256}$

Surds and their Applications

An irrational radical with rational radicand is called a surd. For example: $\sqrt{7}, \sqrt{2}, \sqrt[3]{11}$ are surds but $\sqrt{\pi}, \sqrt{e}$ are not surds.

The different type of surds are as follow:

(i) A surd that contains a single term is called a monomial e.g., $\sqrt{5}, \sqrt{7}$

(ii) A surd that contains the sum of two monomial surds is called a binomial surd e.g., etc.

(iii) $\sqrt{a}+\sqrt{b}$ and $\sqrt{a}-\sqrt{b}$ are called conjugate surds of each other.

Rationalization of denominator

To rationalize a denominator of the form $a + b\sqrt{x}$ or $a - b\sqrt{x}$, we multiply both the numerator and denominator by the conjugate factor.

Example 8: Rationalize the denominator of

(i) $\frac{3}{\sqrt{5}+\sqrt{2}}$ 09301045 (ii) $\frac{3}{\sqrt{5}-\sqrt{3}}$ 09301046

Solution (i):

$$\frac{3}{\sqrt{5}+\sqrt{2}} = \frac{3}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$$= \frac{3(\sqrt{5}-\sqrt{2})}{(\sqrt{5})^2-(\sqrt{2})^2}$$

$$= \frac{3\sqrt{5}-\sqrt{2}}{5-2}$$

$$= \frac{3(\sqrt{5}-\sqrt{2})}{3}$$

$$= \sqrt{5}-\sqrt{2}$$

Solution (ii)

$$\frac{3}{\sqrt{5}-\sqrt{3}} = \frac{3}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$= \frac{3\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2} = \frac{3\left(\sqrt{5}+\sqrt{3}\right)}{5-3} = \frac{3\left(\sqrt{5}+\sqrt{3}\right)}{2}$$

Exercise 1.2

Q.1 Rationalize the denominator of following:

(i) $\frac{13}{4+\sqrt{3}}$ 09301047

Solution

$$\frac{13}{4+\sqrt{3}}$$

Multiplying and dividing by $4 - \sqrt{3}$

$$= \frac{13}{\left(4+\sqrt{3}\right)} \times \frac{\left(4-\sqrt{3}\right)}{\left(4-\sqrt{3}\right)}$$

$$= \frac{13\left(4-\sqrt{3}\right)}{(4)^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{13\left(4-\sqrt{3}\right)}{16-3}$$

$$= \frac{13\left(4-\sqrt{3}\right)}{13}$$

$$= 4-\sqrt{3}$$

(ii) $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}$ 09301048

Solution

$$\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}$$

Multiplying and dividing by $\sqrt{3}$

$$= \frac{\left(\sqrt{2}+\sqrt{5}\right)}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{\sqrt{2}\times\sqrt{3}+\sqrt{5}\times\sqrt{3}}{\left(\sqrt{3}\right)^2}$$

$$= \frac{\sqrt{6}+\sqrt{15}}{3}$$

(iii) $\frac{\sqrt{2}-1}{\sqrt{5}}$ 09301049

Solution

$$\frac{\sqrt{2}-1}{\sqrt{5}}$$

Multiplying and dividing by "$\sqrt{5}$"

$$= \frac{\left(\sqrt{2}-1\right)}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$$= \frac{\left(\sqrt{2}-1\right)\sqrt{5}}{\left(\sqrt{5}\right)^2}$$

$$= \frac{\sqrt{5}\left(\sqrt{2}-1\right)}{5}$$

$$= \frac{\sqrt{10}-\sqrt{5}}{5}$$

(iv) $\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$ 09301050

Solution

$$\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$$

Multiplying and dividing by $\left(6-4\sqrt{2}\right)$

$$= \frac{\left(6-4\sqrt{2}\right)}{\left(6+4\sqrt{2}\right)} \times \frac{\left(6-4\sqrt{2}\right)}{6-4\sqrt{2}}$$

$$= \frac{\left(6-4\sqrt{2}\right)^2}{(6)^2-\left(4\sqrt{2}\right)^2} \quad [\because (a-b)^2 = a^2+b^2-2ab]$$

$$= \frac{(6)^2+\left(4\sqrt{2}\right)^2-2(6)\left(4\sqrt{2}\right)}{36-16(2)}$$

$$= \frac{36+16(2)-48\sqrt{2}}{36-32}$$

$$= \frac{36+32-48\sqrt{2}}{4}$$

$$= \frac{68-48\sqrt{2}}{4}$$

$$= \frac{4\left(17-12\sqrt{2}\right)}{4}$$

$$= 17-12\sqrt{2}$$

(v) $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ **09301051**

Solution

$$\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

Multiplying and dividing by $\left(\sqrt{3}-\sqrt{2}\right)$

$$= \frac{\left(\sqrt{3}-\sqrt{2}\right)}{\sqrt{3}+\sqrt{2}} \times \frac{\left(\sqrt{3}-\sqrt{2}\right)}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{\left(\sqrt{3}-\sqrt{2}\right)^2}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2} \quad [\because (a-b)^2 = a^2+b^2-2ab]$$

$$= \frac{\left(\sqrt{3}\right)^2+\left(\sqrt{2}\right)^2-2\left(\sqrt{3}\right)\left(\sqrt{2}\right)}{3-2}$$

$$= \frac{3+2-2\sqrt{6}}{1}$$

$$= \frac{5-2\sqrt{6}}{1}$$

$$= 5-2\sqrt{6}$$

vi) $\dfrac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}}$

Solution

$$\frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}}$$

3	27
3	9
3	3
	1

$$= \frac{4\sqrt{3}\left(\sqrt{7}-\sqrt{5}\right)}{\left(\sqrt{7}\right)^2-\left(\sqrt{5}\right)^2}$$

$$= \frac{4\sqrt{3}\left(\sqrt{7}-\sqrt{5}\right)}{7-5}$$

$$= \frac{{}^{2}\not{4}\sqrt{3}\left(\sqrt{7}-\sqrt{5}\right)}{\not{2}}$$

$$= 2\sqrt{3}\left(\sqrt{7}-\sqrt{5}\right)$$

Q.2 Simplify the following

(i) $\left(\dfrac{81}{16}\right)^{-\frac{3}{4}}$ **09301052**

Solution

$$\because \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^{m}$$

3	81
3	27
3	9
3	3
	1

$$\left(\frac{81}{16}\right)^{-\frac{3}{4}}$$

$$= \left(\frac{16}{81}\right)^{\frac{3}{4}}$$

2	16
2	8
2	4
2	2
	1

$$= \left(\frac{2^4}{3^4}\right)^{\frac{3}{4}}$$

$$= \frac{2^{4\times\frac{3}{4}}}{3^{4\times\frac{3}{4}}}$$

$$= \frac{2^3}{3^3} = \frac{2\times2\times2}{3\times3\times3}$$

$$= \frac{8}{27}$$

(ii) $\left(\dfrac{3}{4}\right)^{-2} \div \left(\dfrac{4}{9}\right)^{3} \times \dfrac{16}{27}$ **09301053**

Solution

$$\left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{9}\right)^{3} \times \frac{16}{27}$$

$$= \left(\frac{4}{3}\right)^{2} \times \left(\frac{9}{4}\right)^{3} \times \frac{2^4}{3^3}$$

$$= \left(\frac{2^2}{3}\right)^2 \times \left(\frac{3^2}{2^2}\right)^3 \times \frac{2^4}{3^3}$$

$$= \frac{2^4}{3^2} \times \frac{3^6}{2^6} \times \frac{2^4}{3^3}$$

$$= \frac{2^{4+4} \times 3^6}{2^6 \times 3^{2+3}}$$

$$= \frac{2^8 \times 3^6}{2^6 \times 3^5}$$

$$= 2^8 \times 3^6 \times 2^{-6} \times 3^{-5}$$

$$= 2^{8-6} \times 3^{6-5}$$

$$= 2^2 \times 3^1$$

$$= 4 \times 3$$

$$= 12$$

(iii) $\mathbf{(0.027)^{-\frac{1}{3}}}$ **09301054**

Solution

$$(0.027)^{-\frac{1}{3}}$$

$$= \left(\frac{27}{1000}\right)^{-\frac{1}{3}}$$

$$= \left(\frac{1000}{27}\right)^{\frac{1}{3}} \qquad \because \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$$

$$= \left(\frac{10^3}{3^3}\right)^{\frac{1}{3}}$$

$$= \frac{10^{3 \times \frac{1}{3}}}{3^{3 \times \frac{1}{3}}}$$

$$= \frac{10}{3} = 3\frac{1}{3}$$

(iv) $\sqrt[7]{\dfrac{x^{14} \times \mathbf{y}^{21} \times \mathbf{z}^{35}}{\mathbf{y}^{14}\mathbf{z}^{7}}}$ **09301055**

Solution:

$$\sqrt[7]{\frac{x^{14} \times y^{21} \times z^{35}}{y^{14}z^7}}$$

$$\sqrt[7]{x^{14} \times y^{21} \times z^{35} \times y^{-14} \times z^{-7}}$$

$$= \sqrt[7]{x^{14} \times y^{21-14} \times z^{35-7}}$$

$$= \left(x^{14} \times y^7 \times z^{28}\right)^{\frac{1}{7}}$$

$$= x^{14 \times \frac{1}{7}} \times y^{7 \times \frac{1}{7}} \times z^{28 \times \frac{1}{7}}$$

$$= x^2 y z^4$$

(v) $\mathbf{\dfrac{5 \times (25)^{n+1} - 25 \times (5)^{2n}}{5 \times (5)^{2n+3} - (25)^{n+1}}}$ **09301056**

Solution

$$\frac{5 \times (25)^{n+1} - 25 \times (5)^{2n}}{5 \times (5)^{2n+3} - (25)^{n+1}}$$

$$= \frac{5 \cdot (5^2)^{n+1} - (5^2)\left(5^{2n}\right)}{5 \cdot (5)^{2n+3} - (5^2)^{n+1}}$$

$$= \frac{5 \times (5^{2n+2}) - 5^{2n+2}}{5 \times (5^{2n+2} \times 5^1) - 5^{2n+2}}$$

$$= \frac{5 \times 5^{2n+2} - 5^{2n+2}}{5 \times 5^{2n+3} - 5^{2n+2}}$$

$$= \frac{5 \times 5^{2n+2} - 5^{2n+2}}{5 \times 5^{2n+2} \times 5 - 5^{2n+2}}$$

$$= \frac{5^{2n+2}(5-1)}{5^{2n+2} \times 25 - 5^{2n+2}}$$

$$= \frac{5^{2n+2}(4)}{5^{2n+2}(25-1)}$$

$$= \frac{4}{24}$$

$$= \frac{1}{6}$$

(vi) $\mathbf{\dfrac{(16)^{x+1} + 20(4^{2x})}{2^{x-3} \times 8^{x+2}}}$ **09301057**

Solution

$$\frac{(16)^{x+1} + 20(4^{2x})}{2^{x-3} \times 8^{x+2}}$$

$$= \frac{\left(2^4\right)^{x+1} + \left(5 \times 2^2\right)\left(2^2\right)^{2x}}{2^{x-3} \times \left(2^3\right)^{x+2}}$$

$$= \frac{2^{4x+4} + \left(5 \times 2^2\right)\left(2^{4x}\right)}{2^{x-3} \times 2^{3x+6}}$$

$$= \frac{2^{4x+4} + 5 \times 2^{4x+2}}{2^{3x+6+x-3}}$$

$$= \frac{2^{4x+4} + 5 \times 2^{4x+2}}{2^{4x+3}}$$

$$= \frac{\left[2 \times 2 \times 2 \times 2 + 20\right]}{2 \times 2 \times 2}$$

$$= \frac{2^{4x+2} \times 2^2 \times 5 \times 2^{4x+2}}{2^{4x+2} \times 2^1}$$

$$= \frac{2^{4x+2} \times \left(2^2 + 5\right)}{2^{4x+2} \times (2)}$$

$= \dfrac{4+5}{2}$

$= \dfrac{9}{2}$

(vii) $\left(64^{-\frac{2}{3}} \div (9)^{\frac{-3}{2}}\right)$ **09301058**

Solution

$(64)^{-\frac{2}{3}} \div (9)^{-\frac{3}{2}}$

$= \dfrac{(64)^{-\frac{2}{3}}}{(9)^{-\frac{3}{2}}}$

$= \dfrac{9^{\frac{3}{2}}}{64^{\frac{2}{3}}} \qquad \left(\because \dfrac{a^{-m}}{b^{-n}} = \dfrac{b^n}{a^m}\right)$

$= \dfrac{\left(3^2\right)^{\frac{3}{2}}}{\left(2^6\right)^{\frac{2}{3}}}$

$= \dfrac{3^{2\times\frac{3}{2}}}{2^{6\times\frac{2}{3}}}$

$= \dfrac{3^3}{2^{2\times 2}}$

$= \dfrac{27}{2^4}$

$= \dfrac{27}{16}$

(viii) $\dfrac{3^n \times 9^{n+1}}{3^{n-1} \times 9^{n-1}}$ **09301059**

Solution

$\dfrac{3^n \times 9^{n+1}}{3^{n-1} \times 9^{n-1}}$

$= \dfrac{3^n \times \left(3^2\right)^{n+1}}{3^{n-1} \times \left(3^2\right)^{n-1}}$

$= \dfrac{3^n \times 3^{2n+2}}{3^{n-1} \times 3^{2n-2}}$

$= \dfrac{3^{n+2n+2}}{3^{n-1+2n-2}}$

$= \dfrac{3^{3n+2}}{3^{3n-3}}$

$= 3^{3n+2} \times 3^{-3n+3}$

$= 3^{3n+2-3n+3} \qquad \left(\because \dfrac{a^m}{a^n} = a^{m-n}\right)$

$=3^5$

$3\times3\times3\times3\times3$

$=243$

(ix) $\dfrac{5^{n+3} - 6.5^{n+1}}{9\times5^n - 2^2\times5^n}$ **09301060**

Solution

$\dfrac{5^{n+3} - 6.5^{n+1}}{9\times5^n - 2^2\times5^n}$

$= \dfrac{5^n\times5^3 - 6.5^n.5^1}{9\times5^n - 2^2\times5^n}$

$= \dfrac{5^n\left(5^3 - 6\times5\right)}{5^n\left(9-2^2\right)}$

$= \dfrac{5^3 - 6\times5}{9-2^2}$

$= \dfrac{125-30}{9-4}$

$= \dfrac{95}{5}$

$= 19$

Q.3 If $x = 3 + \sqrt{8}$ then find the value of

(i) $x + \dfrac{1}{x}$ (ii) $x - \dfrac{1}{x}$ (iii) $x^2 + \dfrac{1}{x^2}$

(iv) $x^2 - \dfrac{1}{x^2}$ (v) $x^4 + \dfrac{1}{x^4}$ (vi) $\left(x - \dfrac{1}{x}\right)^2$

Solutions

$x = 3+\sqrt{8}$ __________(i)

$\dfrac{1}{x} = \dfrac{1}{3+\sqrt{8}}$

$\dfrac{1}{x} = \dfrac{1}{\left(3+\sqrt{8}\right)} \times \dfrac{\left(3-\sqrt{8}\right)}{\left(3-\sqrt{8}\right)}$

$\dfrac{1}{x} = \dfrac{3-\sqrt{8}}{(3)^2 - \left(\sqrt{8}\right)^2}$

$\dfrac{1}{x} = \dfrac{3-\sqrt{8}}{9-8}$

$\dfrac{1}{x} = \dfrac{3-\sqrt{8}}{1}$

$\dfrac{1}{x} = 3-\sqrt{8}$ _______ (ii)

(i) Finding $x+\frac{1}{x}$ 09301061

Adding eq. (i) and (ii)

$$x+\frac{1}{x}=3+\sqrt{8}+3-\sqrt{8}$$

$$x+\frac{1}{x}=6\_\_\_\_\_\_\_\text{ (iii)}$$

(ii) Finding $x-\frac{1}{x}$ 09301062

Subtracting eq.(i) from (ii)

$$x-\frac{1}{x}=\left(3+\sqrt{8}\right)-\left(3-\sqrt{8}\right)$$

$$x-\frac{1}{x}=3+\sqrt{8}-3+\sqrt{8}$$

$$x-\frac{1}{x}=2\sqrt{8}\text{ ---------------(iv)}$$

(iii) Finding $x^2+\frac{1}{x^2}$ 09301063

Taking square of eq. (iii)

$$\left(x+\frac{1}{x}\right)^2=(6)^2$$

$$(x)^2+\left(\frac{1}{x}\right)^2+2(x)\left(\frac{1}{x}\right)=36$$

$$x^2+\frac{1}{x^2}=36-2$$

$$x^2+\frac{1}{x^2}=34\text{ -----------(v)}$$

(iv)Finding $x^2-\frac{1}{x^2}$

We know that

$$x^2-\frac{1}{x^2}=(x)^2-\left(\frac{1}{x}\right)^2$$

$$x^2-\frac{1}{x^2}=\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$$

Putting the values from (iii) and (iv)

$$x^2-\frac{1}{x^2}=(6)\left(2\sqrt{8}\right)=12\sqrt{8}$$

(v) Finding $x^4+\frac{1}{x^4}$

Taking square of equation (v)

$$\left(x^2+\frac{1}{x^2}\right)^2=(34)^2$$

$$\left(x^2\right)^2+\left(\frac{1}{x^2}\right)^2+2\left(x^2\right)\left(\frac{1}{x^2}\right)=1156$$

$$x^4+\frac{1}{x^4}+2=1156$$

$$x^4+\frac{1}{x^4}=1156-2$$

$$x^4+\frac{1}{x^4}=1154$$

(vi)Finding $\left(x-\frac{1}{x}\right)^2$

Taking square of equation (iv)

$$\left(x-\frac{1}{x}\right)^2=\left(2\sqrt{8}\right)^2$$

$$\left(x-\frac{1}{x}\right)^2=(2)^2\left(\sqrt{8}\right)^2$$

$$\left(x-\frac{1}{x}\right)^2=4(8)$$

$$\left(x-\frac{1}{x}\right)^2=32$$

Q.4 Find the rational numbers p and q such that $\frac{8-3\sqrt{2}}{4+3\sqrt{2}}=p+q\sqrt{2}$ 09301064

Solution

$$\mathbf{p+q\sqrt{2}=\frac{8-3\sqrt{2}}{4+3\sqrt{2}}}$$

$$\text{p}+\text{q}\sqrt{2}=\frac{\left(8-3\sqrt{2}\right)}{\left(4+3\sqrt{2}\right)}\times\frac{\left(4-3\sqrt{2}\right)}{\left(4-3\sqrt{3}\right)}$$

$$=\frac{32-24\sqrt{2}-12\sqrt{2}+\left(3\sqrt{2}\right)^2}{(4)^2-\left(3\sqrt{2}\right)^2}$$

$$=\frac{32-36\sqrt{2}+9(2)}{16-9(2)}$$

$$=\frac{32-36\sqrt{2}+18}{16-18}$$

$$=\frac{50-36\sqrt{2}}{-2}$$

$$= \frac{50}{-2} - \frac{36\sqrt{2}}{-2}$$

$p+q\sqrt{2} = -25 + 18\sqrt{2}$

By comparing both sides.

$\Rightarrow$ p = –25, q = 18

5. Simplify the following:

(i) $\dfrac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}}$ **09301065**

Solution

3	243
3	81
3	27
3	9
3	3
	1

$$\frac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}}$$

$$= \frac{\left(5^2\right)^{\frac{3}{2}} \times \left(3^5\right)^{\frac{3}{5}}}{\left(2^4\right)^{\frac{5}{4}} \times \left(2^3\right)^{\frac{4}{3}}}$$

$$= \frac{\left(5^{\cancel{2}}\right)^{\frac{3}{\cancel{2}}} \times \left(3^{\cancel{5}}\right)^{\frac{3}{\cancel{5}}}}{\left(2^{\cancel{4}}\right)^{\frac{5}{\cancel{4}}} \times \left(2^{\cancel{3}}\right)^{\frac{4}{\cancel{3}}}}$$

$$= \frac{5^3 \times 3^3}{2^5 \times 2^4}$$

$$= \frac{(5\times 3)^3}{2^{5+4}} \qquad \left(\because a^m \times a^n = a^{m+n}\right)$$

$$= \frac{(15)^3}{2^9}$$

$$= \frac{15\times 15\times 15}{2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2}$$

$$= \frac{3375}{512}$$

(ii) $\dfrac{54 \times \sqrt[3]{(27)^{2x}}}{9^{x+1} + 216(3^{2x-1})}$ **09301066**

Solution

2	54
3	27
3	9
3	3
	1

2	216
2	108
2	54
3	27
3	9
3	3
	1

$$\frac{54 \times \sqrt[3]{(27)^{2x}}}{9^{x+1} + 216(3^{2x-1})}$$

$$= \frac{\left(2\times 3^3\right) \times \sqrt[3]{\left(3^3\right)^{2x}}}{\left(3^2\right)^{x+1} + 2^3 \times 3^3\left(3^{2x-1}\right)}$$

$$= \frac{\left(2\times 3^3\right) \times \sqrt[3]{\left(3^{2x}\right)^3}}{\ldots}$$

$$\left(\because \sqrt[m]{a^m} = a\right)$$

$$= \frac{2\times 3^3 \times 3^{2x}}{3^{2x+2} + 2^3 \times 3^{2x+2}}$$

$$= \frac{2\times 3^{2x+3}}{3^{2x+2}\left(1+2^3\right)}$$

$$= \frac{2\times 3^{\cancel{2x}+3-\cancel{2x}-2}}{(1+8)} \qquad \left(\because \frac{a^m}{a^n} = a^{m-n}\right)$$

$$= \frac{2\times 3^1}{9}$$

$$= \frac{6}{9}$$

$$= \frac{2}{3}$$

(iii) $\sqrt{\dfrac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{\frac{-3}{2}}}}$ **09301067**

Solution

$$\sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{\frac{-3}{2}}}}$$

$$= \sqrt{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}} \times (0.04)^{\frac{3}{2}}}$$

2	216
2	108
2	54
3	27
3	9
3	3
	1

$$= \sqrt{\left(2^3\times 3^3\right)^{\frac{2}{3}} \times \left(5^2\right)^{\frac{1}{2}} \times \left(\frac{4}{100}\right)^{\frac{3}{2}}}$$

$$= \sqrt{2^{3\times\frac{2}{3}} \times 3^{3\times\frac{2}{3}} \times 5^{2\times\frac{1}{2}} \times \left(\frac{1}{25}\right)^{\frac{3}{2}}}$$

$$= \sqrt{2^2 \times 3^2 \times 5^1 \times \frac{1}{\left(5^2\right)^{\frac{3}{2}}}}$$

$$= \sqrt{(2\times 3)^2 \times 5^1 \times \frac{1}{5^3}}$$

$$= \sqrt{(6)^2 \times \frac{1}{5^{3-1}}}$$

$$= \sqrt{\frac{6^2}{5^2}}$$

$$= \sqrt{\left(\frac{6}{5}\right)^2}$$

$$= \frac{6}{\ldots}$$

(iv) $\left(a^{\frac{1}{3}}+b^{\frac{2}{3}}\right)\times\left(a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{2}{3}}+b^{\frac{4}{3}}\right)$ **09301068**

Solution

$$\left(a^{\frac{1}{3}}+b^{\frac{2}{3}}\right)\times\left(a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{2}{3}}+b^{\frac{4}{3}}\right)$$

$$=\left(a^{\frac{1}{3}}+b^{\frac{2}{3}}\right)\times\left[\left(a^{\frac{1}{3}}\right)^2-\left(a^{\frac{1}{3}}\right)\left(b^{\frac{2}{3}}\right)+\left(b^{\frac{2}{3}}\right)^2\right]$$

$$\because (x+y)(x^2-xy+y^2)=x^3+y^3$$

$$\therefore = \left(a^{\frac{1}{3}}\right)^3+\left(b^{\frac{2}{3}}\right)^3$$

$$= a^{\frac{1}{3}\times 3}+b^{\frac{2}{3}\times 3}$$

$$= a+b^2$$

Applications of Real Numbers in Daily Life:

Example 9: The sum of two real numbers is 8, and their difference is 2. Find the numbers. **09301069**

Solution

Let a and b be two real numbers then

$a+b=8$ ---------------(i)

$a-b=2$ ---------------(ii)

Add eq. (i) and eq. (ii)

$2a=10$

$\Rightarrow a=5$

Put it in eq. (i)

$\Rightarrow 5-b=2$

$\Rightarrow -b=2-5$

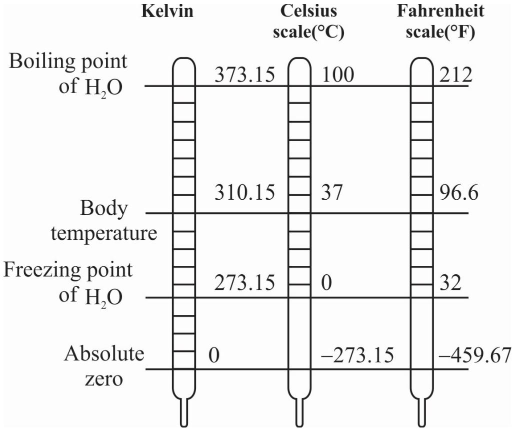
$\Rightarrow -b=-3$

$\Rightarrow b=3$

So, 5 and 3 are required real numbers

Temperature Conversions

In the figure, three types of thermometers are shown.



We can convert three temperature scales Celsius, Fahrenheit and kelvin with each other.

Conversion formulae are given below:

(i) K = °C +273 **09301070**

(ii) $°C = \frac{5}{9}(F-32)°$ **09301071**

(iii) $°F = \frac{9°C}{5}+32$ **09301072**

Where K,C and F shows the kelvin, Celsius and Fahrenheit scales respectively.

Example 10: Normal human body temperature is 98.6 F. Convert it into Celsius and kelvin scale. **09301073**

Solution

Given that

°F = 98.6

So Convert it into Celsius scale, we use

$°C = \frac{5}{9}(F-32)°$

$°C = \frac{5}{9}(98.6-32)°$

$°C = \frac{5}{9}(66.6-32)°$

°C = (0.55) (66.6)°

°C = 37°

Hence, normal human body temperature at Celsius scale is 37°

Now, we convert it into Kelvin scale

K = C +273°

K = 37° + 273°

K = 310 Kelvin

Profit and Loss:

The profit and loss can be calculated by the following formula.

(i)Profit = Selling price – Cost Price

Profit = SP– CP

$\text{Profit \%} = \left(\frac{\text{Profit}}{CP}\times 100\right)\%$

(ii)Loss = Cost Price – Selling Price

Loss = CP – SP

$\text{Loss \%} = \left(\frac{loss}{CP}\times 100\right)\%$

Example 11: Hamail purchased a bicycle for Rs. 6590 and sold it for Rs. 6850. Find the profit percentage. **09301074**

Solution

Cost Price = CP = Rs. 6590

Selling Price = SP = Rs. 6850

Profit = SP – CP

= 6850 – 6590

= Rs 260

Now, we find the profit percentage.

$$\text{Profit \%} = \left(\frac{profit}{CP}\times 100\right)\%$$
$$= \left(\frac{260\times 100}{6590}\right)\%$$
$$= 3.94\% \approx 4\%$$

Example 12: Umair bought a book for Rs. 850 and sold it for Rs. 720. What was his loss percentage? **09301075**

Solution

Cost price of book = CP = Rs. 850
Selling price of book = SP = Rs. 720

$$\text{Loss} = CP - SP$$
$$= 850 - 720$$
$$= \text{Rs. } 130$$

$$\text{Loss percentage} = \left(\frac{Loss}{CP}\times 100\right)\%$$
$$= \left(\frac{130}{850}\times 100\right)\%$$
$$= 15.29\%$$

Example 13: Mr. Saleem, Nadeem, and Tanveer earned a profit of Rs. 450,000 from a business. If their investments in the business are the ratio 4: 7: 14, find each person's profit. **09301076**

Solution

Profit earned = Rs. 450,000
Given ratios = 4: 7: 14
Sum of ratio = 4 + 7 + 14 = 25

$$\text{Saleem earned profit} = \frac{4}{25}\times 450{,}000$$
$$= \text{Rs. } 72{,}000$$

$$\text{Nadeem earned profit} = \frac{7}{25}\times 450{,}000$$
$$= \text{Rs. } 126{,}000$$

$$\text{Tanveer earned profit} = \frac{14}{25}\times 450{,}000$$
$$= \text{Rs. } 252{,}000$$

Example 14: If the simple profit on Rs. 6400 for 12 years is Rs. 3840. Find the rate of profit. **09301077**

Solution

Principal = Rs. 6400
Simple profit = Rs. 3840
Time = 12 years

To find the rate we use the following formula

$$\text{Rate} = \frac{amount\ of\ profit\times 100}{time\times principal}$$
$$= \frac{3840\times 100}{12\times 6400}$$
$$= 5\%$$

Thus, rate of profit is 5%.

Exercise 1.3

Q.1 The sum of three consecutive integers is forty-two, find three integers. **09301078**

Solution

Let x, $x+1$, $x+2$ be three consecutive integers

By condition

$x + (x+1) + (x+2) = 42$

$x + x + 1 + x + 2 = 42$

$3x + 3 = 42$

$3x = 42 - 3$

$3x = 39$

$3x = \frac{39}{3}$

$x = 13$

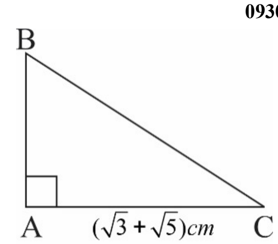
Now the 1st integer = $x = 13$

2nd integer = $x+1 = 13+1 = 14$

3rd integer = $x+2 = 13+2 = 15$

Thus 13,14 and 15 are required three consecutive integers.

Q.2 The diagram shows right angled Δ ABC in which the length of AC is $(\sqrt{3}+\sqrt{5})$ cm. The area of Δ ABC is $(1+\sqrt{15})$ cm², Find the length AB in the form $(a\sqrt{3}+b\sqrt{5})$ cm where a and b are integers. **09301079**



Solution

Given

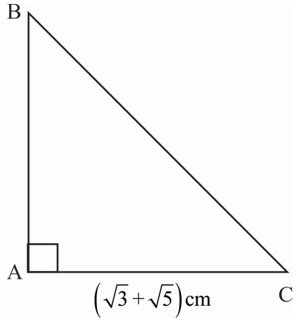
$m\overline{AC} = \left(\sqrt{3}+\sqrt{5}\right)\text{cm}$

Area of $\Delta ABC = \left(1+\sqrt{15}\right)\text{cm}^2$

To find = $m\overline{AB} = ?$

We know that, Area of $\Delta = \frac{1}{2}(b\times h)$

Area of $\Delta ABC = \frac{1}{2}\left(m\overline{AC}\times m\overline{AB}\right)$



$1+\sqrt{15}=\frac{1}{2}\left[\left(\sqrt{3}+\sqrt{5}\right)\times m\overline{AB}\right]$

$\frac{2\left(1+\sqrt{15}\right)}{\sqrt{3}+\sqrt{5}}=m\overline{AB}$

$\Rightarrow$ Multiplying and dividing by $\left(\sqrt{3}-\sqrt{5}\right)$

$\Rightarrow\ m\overline{AB}=\frac{\left(2+2\sqrt{15}\right)}{\left(\sqrt{3}+\sqrt{5}\right)}\times\frac{\left(\sqrt{3}-\sqrt{5}\right)}{\sqrt{3}-\sqrt{5}}$

$=\ \frac{2\sqrt{3}-2\sqrt{5}+2\sqrt{15}\times\sqrt{3}-2\sqrt{15}\times\sqrt{5}}{\left(\sqrt{3}\right)^2-\left(\sqrt{5}\right)^2}$

[$\because\ a^2-b^2=(a+b)(a-b)$

$=\ \frac{2\sqrt{3}-2\sqrt{5}+2\sqrt{45}-2\sqrt{75}}{3-5}$

$=\ \frac{2\left(\sqrt{3}-\sqrt{5}+\sqrt{9\times5}-\sqrt{25\times3}\right)}{-2}$

$=\ \frac{\sqrt{3}-\sqrt{5}+3\sqrt{5}-5\sqrt{3}}{-1}$

$=\ \frac{-4\sqrt{3}+2\sqrt{5}}{-1}$

$=\ 4\sqrt{3}-2\sqrt{5}$

Thus $m\overline{AB}=\left(4\sqrt{3}-2\sqrt{5}\right)$ cm.

Q.3 A rectangle has sides of length $(2+\sqrt{18})m$ and $(5-\frac{4}{\sqrt{2}})m$. Express the area of the rectangle in the form $a+b\sqrt{2}$ where a and b are integers. 09301080

Solution

Let length of rectangle = L = $\left(2+\sqrt{18}\right)$m

Width of rectangle = W = $\left(5-\frac{4}{\sqrt{2}}\right)$m

We know that

Area of Rectangle = A = L × W

A $=\left(2+\sqrt{18}\right)\times\left(5-\frac{4}{\sqrt{2}}\right)$

A $=\left(2+\sqrt{9\times2}\right)\times\left(5-\frac{2\times2}{\sqrt{2}}\right)$

A $=\left(2+3\sqrt{2}\right)\times\left(5-2\sqrt{2}\right)$ $\left(\because\frac{a}{\sqrt{a}}=\sqrt{a}\right)$

A $=10-\ 4\sqrt{2}+15\sqrt{2}-(3\sqrt{2})(2\sqrt{2})$

A $=10+11\sqrt{2}-6\sqrt{4}$

A $=10+11\sqrt{2}-6(2)$

A $=10+11\sqrt{2}-12$

A $=-2+11\sqrt{2}$

Thus area of rectangle is $\left(-2+11\sqrt{2}\right)m^2$

Q.4 Find two numbers whose sum is 68 and whose difference is 22. 09301081

Solution

Sum = 68

Difference = 22

Let x and y be required numbers

By given conditions:

Sum: $x+y=68$ _________ (i)

Diff. $x-y=22$ _________ (ii)

Adding eq. (i) and (ii),

$$\begin{array}{l} x+\not{y}=68 \\ \underline{x-\not{y}=22} \\ 2x\quad =90 \end{array}$$

$x=\frac{90}{2}$

$x=45$

Put it in eq. (i),

$x+y=68$

$45+y=68$

$y=68-45$

$y=23$

Thus required numbers are 45 and 23.

Q.5 The weather in Lahore was unusually warm during the summer of 2024. The TV news reported temperatures as high as 48°C. By using the formula, $\left(°F=\frac{9}{5}°C+32\right)$ find the temperature as Fahrenheit scale. 09301082

Solution

Temperature in degree centigrade = 48°C

formula: °F = $\frac{9}{5}$°C+32

Temperature in Fahrenheit,

$°F = \frac{9}{5} \times 48 + 32$

$°F = 9 \times 9.6 + 32$

$°F = 86.4 + 32$

$°F = 118.4$

Q.6 The sum of the ages of the father and son is 72. Six years ago the father's age was 2 times the age of the son. What was Son's age six years ago? 09301083

Solution

Let father's age = x

Son's age = y

By condition: $x+y = 72$ _________ (i)

Six year ago,

Father's age = $x-6$

Son's age = $y-6$

By condition:

Father's age = 2 times the son's age

$(x-6) = 2(y-6)$

$x-6 = 2y-12$

$x-2y = 6-12$

$x-2y = -6$_________________ (ii)

Subtracting eq. (ii) from (i)

$$\begin{array}{r} \cancel{x} + \quad y = 72 \\ \pm\cancel{x} \mp 2y = \mp 6 \\ \hline 3y = 78 \\ y = \frac{78}{3} \\ \boxed{y = 26} \end{array}$$

Put y = 26 in eq.(i)

$x + 26 = 72$

$x = 72 - 26$

$\boxed{x = 46}$

Six years ago,

Son's age = y – 6= 26–6 = 20 years

Father's age = x – 6= 46–6= 40 years

Q.7 Mirha buys a toy for Rs. 1520. What will the selling price be to get a 15% profit? 009301084

Solution

The cost price = CP = Rs.1520

Profit rate = 15%

We know that

Profit = 15% of CP

Profit $= \frac{15}{100} \times 1520$

$= 0.15 \times 1520$

= Rs.228

Spelling Price = CP + Profit

= Rs.1520 + Rs.228

= Rs.1748

Thus to get 15% profit the spelling price must be Rs.1748.

Q.8 The annual income of Tayab is Rs. 960,000, while the exempted amount is Rs. 130,000. How much tax would he have to pay at the rate of 0.75%. 09301085

Solution

Annual income = Rs.9,60,000/-

Exempted amount = Rs.130,000/-

Tax rate = 0.75%

We know that

Taxable income

= Annual income – exempted amount

Taxable income = 960,000 – 130,000

Taxable income = 830,000

Tax amount = 0.75% of Taxable income

$= \frac{0.75}{100} \times 830,000$

$= \frac{75}{100 \times 100} \times 830,000$

$= 75 \times 83$

= Rs.6,225

Thus Tayab will pay tax of Rs.6,225.

Q.9 Find the compound markup on Rs. 3,75,000 for one year at the rate of 14% compounded markup annually. 09301086

Solution

Principal amount = P = Rs.375,000/-

Time, = t = 1year

Rate , R = 14%

We know that

Compound markup = P×T×R

= 375,000×1×14%

$= 375,000 \times \frac{14}{100}$

$= 3,750 \times 14$

= Rs. 52,500/-

Thus compound mark up is Rs.52,500/-

Review Exercise – 1

Q.1 Choose the correct option.

i. $\sqrt{7}$ is: 09301087
(a) Integer
(b) Rational number
(c) Irrational number
(d) Natural number

ii. π and e are: 09301088
(a) Natural number
(b) Integers
(c) Rational number
(d) Irrational number

iii. If n is not a perfect square then $\sqrt{n}$ is: 09301089
(a) Rational number
(b) Natural number
(c) Integer
(d) Irrational number

iv. $\sqrt{3}+\sqrt{5}$ is: 09301090
(a) Whole number
(b) Integer
(c) Rational number
(d) Irrational number

v. For all $x \in R,\ x = x$ is called: 09301091
(a) Reflexive property
(b) Transitive number
(c) Symmetric property
(d) Trichotomy property

vi. Let $a,b,c \in R$ then $a > b$ and $b > c$ $\Rightarrow a > c$ is called __________property. 09301092
(a) Trichotomy (b) Transitive
(c) Additive (d) Multiplicative

vii. $2^x \times 8^x = 64$ then $x =$ 09301093
(a) $\frac{3}{2}$ (b) $\frac{3}{4}$
(c) $\frac{5}{6}$ (d) $\frac{2}{3}$

viii. Let $a,b,\in R$ then $a = b$ and $b = a$ is called __________property. 09301094
(a) Reflexive (b) Symmetric
(c) Transitive (d) Additive

ix. $\sqrt{75}+\sqrt{27} =$ 09301095
(a) $\sqrt{102}$ (b) $9\sqrt{3}$
(c) $5\sqrt{3}$ (d) $8\sqrt{3}$

x. The product of $(3+\sqrt{5})(3-\sqrt{5})$ is: 09301096
(a) Prime number
(b) odd number
(c) Irrational number
(d) Rational number

Answer Key

i	c	**ii**	d	**iii**	d	**iv**	d	**v**	a
vi	b	**vii**	a	**viii**	b	**ix**	d	**x**	d

Multiple Choice Questions (Additional)

History of Real numbers:

1. Which number system was used by the Sumerians? 09301097
(a) Decimal (b) hexadecimal
(c) Sexagesimal (d) Binary

2. The sexagesimal system is a number system with the base: 09301098
(a) 2 (b) 10
(c) 16 (d) 60

3. Which number system was used by the Egyptians? 09301099
(a) Decimal (b) hexadecimal
(c) Sexagesimal (d) Binary

4. How many letters are used in Roman numeral system? 09301100
(a) 3 (b) 5
(c) 7 (d) 10

5. In Roman counting the letter "L" represents the number: 09301101
(a) 10 (b) 50
(c) 100 (d) 500

6. The invention of zero is attributed to: 09301102
(a) Arabs (b) Egyptians
(c) Sumerians (d) Indians

7. Which number system is known as Indo-Arabic numerals? 09301103
(a) Decimal (b) hexadecimal
(c) Sexagesimal (d) Binary

8. Who did introduce the numerals (0-9) to Europe? 09301104
(a) Arabs (b) Egyptians

(c) Sumerians (d) Indian

9. Which of the following is one of the modern number systems? 09301105
   (a) Roman Numerals
   (b) Egyptians numerals
   (c) Sexagesimal system
   (d) hexadecimal system

Real numbers:

10. $Q \cup Q' =$ ____ 09301106
    (a) Q' (b) Q
    (c) R (d) ϕ
11. $Q \cap Q' =$ ____ 09301107
    (a) Q' (b) Q
    (c) R (d) ϕ
12. Q and Q′are ______sets. 09301108
    (a) disjoint (b) over lapping
    (c) intersecting (d) supper
13. For each prime number P, $\sqrt{P}$ is an: 09301109
    (a) Irrational (b) Rational
    (c) Real (d) Whole

Properties of real numbers:

14. Name the property of real numbers used in $\pi + (-\pi) = 0$. 09301110
    (a) Additive inverse
    (b) Multiplicative inverse
    (c) Additive identity
    (d) Multiplicative identity
15. Name the property of real numbers used in $\frac{1}{2} \times 1 = \frac{1}{2}$. 09301111
    (a)Additive identity
    (b) Additive Inverse
    (c) Multiplicative identity
    (d) Multiplicative Inverse
16. If $x < y$ and z < 0 then: 09301112
    (a) x z < y z (b) x z > y z
    (c) x z = y z (d) ẍ > y
17. If a, b ∈ R then only one of a = b or a < b or a > b holds is called --- property. 09301113
    (a) Trichotomy (b) Transitive
    (c) Additive (d) Multiplicative

Radical Expressions:

18. In $\sqrt[r]{a}$, the symbol $\sqrt{\ }$ is called: 09301114
    (a) radical sign (b) index
    (c) exponent (d) base
19. In $\sqrt[n]{a^m}$ ‘n’ is called: 09301115
    (a) base (b) radical sign
    (c) index (d) radical
20. $(27x)^{\frac{2}{3}} =$ _____ 09301116
    (a) $\frac{\sqrt[3]{x^2}}{9}$ (b) $\frac{\sqrt{x^3}}{9}$
    (c) $\frac{\sqrt[3]{x^2}}{8}$ (d) $9\sqrt[3]{x^2}$
21. Write $\sqrt[5]{x}$ in exponential form 09301117
    (a) x (b) x^5
    (c) $x^{\frac{1}{5}}$ (d) $x^{\frac{5}{2}}$
22. Writing $m^{\frac{2}{3}}$ with radical sign we get: 09301118
    (a) $\sqrt[3]{m^2}$ (b) $\sqrt{m^3}$
    (c) $\sqrt[2]{m^3}$ (d) $\sqrt{m^6}$
23. In $\sqrt[3]{5}$, the radicand is: 09301119
    (a) 3 (b) $\frac{1}{3}$
    (c) 5 (d) 35

Surds:

24. Which of the following is a Surd? 09301120
    (a) $\sqrt{7}$ (b) $\sqrt{9}$
    (c) $\sqrt{\pi}$ (d) $\sqrt{e}$
25. Which of the following is a binomial Surd? 09301121
    (a) $4\sqrt{3}$ (b) $\sqrt{16}$
    (c) $7+\sqrt{\pi}$ (d) $2-\sqrt{3}$
26. A surd which contains a single term is called surd. 09301122
    (a) Monomial (b) Binomial
    (c) Trinomial (d) None
27. Conjugate factor of the Surd $a+b\sqrt{x}$ is: 09301123
    (a) $a+b\sqrt{x}$ (b) $a-b\sqrt{x}$
    (c) $-a-b\sqrt{x}$ (d) $-a+b\sqrt{x}$
28. $(4+\sqrt{2})(4-\sqrt{2})$ is equal to: 09301124
    (a) 14 (b) –14
    (c) 12 (d) 8
29. $\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)(x+y) = ...$ 09301125
    (a) $(x+y)$ (b) $(x-y)$
    (c) $\left(x^2+y^2\right)$ (d) $\left(x^2-y^2\right)$
30. $\frac{1}{2-\sqrt{3}} =$ ____ 09301126
    (a) $2+\sqrt{3}$ (b) $2-\sqrt{3}$
    (c) (d)

Answer Key

1	**c**	2	**d**	3	**a**	4	**c**	5	**b**	6	**d**	7	**a**	8	**a**	9	**d**	10	**c**
11	**d**	12	**a**	13	**a**	14	**a**	15	**c**	16	**b**	17	**a**	18	**a**	19	**c**	20	**d**
21	**c**	22	**a**	23	**c**	24	**a**	25	**d**	26	**a**	27	**b**	28	**a**	29	**d**	30	**a**

Q.2 If $a=\frac{3}{2}, b=\frac{5}{3}$ and $c=\frac{7}{5}$ then verify that

(i) $a(b+c) = ab + ac$ **09301127**

Solution

$a(b+c)=ab+ac$

L.H.S $= a(b+c)$

$= \frac{3}{2}\left(\frac{5}{3}+\frac{7}{5}\right)$

$= \frac{3}{2}\left(\frac{25+21}{15}\right)$

$= \frac{3}{2}\left(\frac{46}{15}\right)$

$= \frac{23}{5}$ _____ (i)

Now, R.H.S $= ab+ac$

$= \frac{3}{2}\times\frac{5}{3}+\frac{3}{2}\times\frac{7}{5}$

$= \frac{5}{2}+\frac{21}{10}$

$= \frac{25+21}{10}$

$= \frac{46}{10}$

$= \frac{23}{5}$ _____ (ii)

From (i) and (ii), L.H.S = R.H.S

Hence a(b=c) = ab + ac

(ii) (a+b) c = ac + bc **09301128**

Solution

$(a+b)c=ac+bc$

L.H.S $= (a+b)c$

$= (a+b)c$

$= \left(\frac{3}{2}+\frac{5}{3}\right)\times\frac{7}{5}$

$= \left(\frac{9+10}{6}\right)\times\frac{7}{5}$

$= \frac{19}{6}\times\frac{7}{5}$

133

$= \frac{3}{2}\times\frac{7}{5}+\frac{5}{3}\times\frac{7}{5}$

$= \frac{21}{10}+\frac{7}{3}$

$= \frac{63+70}{30}$

$= \frac{113}{30}$ ______(ii)

From (i) and (ii), L.H.S = R.H.S

Hence (a+b)c = ac + bc is proved

Q.3 If $a=\frac{4}{3}$, b = $\frac{5}{2}$, c = $\frac{7}{4}$, then verify the associative property of real numbers w.r.t addition and multiplication. **09301129**

Solution

(i) Associative property w.r.t addition.

(a+b) +c = a + (b+c)

L.H.S $= (a+b)+c$

$= \left(\frac{4}{3}+\frac{5}{2}\right)+\frac{7}{4}$

$= \left(\frac{8+15}{6}\right)+\frac{7}{4}$

2	6 –4
2	3–2
3	3–1
	1–1

$= \frac{23}{6}+\frac{7}{4}$

$= \frac{46+21}{12}$

$= \frac{67}{12}$ ________ (i)

Now, $= \frac{4}{3}+\left(\frac{5}{2}+\frac{7}{4}\right)$

$= \frac{4}{3}+\left(\frac{10+7}{4}\right)$

$= \frac{4}{3}+\frac{17}{4}$

$= \frac{16+51}{12}$

$= \frac{67}{12}$ ______ (ii)

From(i) and (ii) L.H.S = R.H.S.

(ii) Associative property w.r.t multiplication.

09301130

$(a\times b)\times c = a\times(b\times c)$

Solution

L.H.S $= (a\times b)\times c$

$= \left(\frac{4}{3}\times\frac{5}{2}\right)\times\frac{7}{4}$

$= \frac{20}{6}\times\frac{7}{4}$

$= \frac{10}{3}\times\frac{7}{4}$

$= \frac{70}{12}$

$= \frac{35}{6}$ ______ (i)

Now, R.H.S $= a\times(b\times c)$

$= \frac{4}{3}\times\left(\frac{5}{2}\times\frac{7}{4}\right)$

$= \frac{4}{3}\times\left(\frac{35}{8}\right)$

$= \frac{{}^{1}\not{4}}{3}\times\left(\frac{35}{\not{8}_{2}}\right)$

$= \frac{35}{6}$ ______ (ii)

From (i) and (ii) L.H.S = R.H.S

Hence, $(a\times b)\times c = a\times(b\times c)$

Q.4 Is 0 a rational number? Explain.

09301131

Solution

Yes, 0 is a rational number.

Explanation

A number in the form $\frac{p}{q}$, p where p, q $\in$z and q $\neq$ 0 is a rational number. The number 0 can be written as $\frac{0}{1},\frac{0}{2},\frac{0}{3},\ldots\ldots$.. Here 0$\in$z and 1,2,3, $\in$z so we can say that 0 is a rational number.

Q.5 State trichotomy property of real numbers. **09301132**

Solution:

For all values of a, b $\in$ R

Either $a > b$ or $a = b$ or $a < b$

This property is called trichotomy property.

Q.6 Find two rational numbers between 4 and 5. **09301133**

Solution

Average of 4 and 5 = $\frac{4+5}{2} = \frac{9}{2}$

Now we find,

Average of $\frac{9}{2}$ and 5 $= \left(\frac{9}{2}+5\right)\div 2$

$= \left(\frac{9+10}{2}\right)\times\frac{1}{2}$

$= \frac{19}{2}\times\frac{1}{2}$

$= \frac{19}{4}$

Thus two rational number between 4 and 5 are $\frac{9}{2}$ and $\frac{19}{4}$.

Q.7 Simplify the following:

(i) $\sqrt[5]{\frac{x^{15}y^{35}}{z^{20}}}$ **09301134**

Solution

$\sqrt[5]{\frac{x^{15}y^{35}}{z^{20}}} = \left(\frac{x^{15}y^{35}}{z^{20}}\right)^{\frac{1}{5}}$

$= \frac{x^{15\times\frac{1}{5}}y^{35\times\frac{1}{5}}}{z^{20\times\frac{1}{5}}}$

$= \frac{x^3.y^7}{z^4}$

(ii) $\sqrt[3]{(27)^{2x}}$ **09301135**

Solution

$\sqrt[3]{(27)^{2x}}$

$= \sqrt[3]{(3^3)^{2x}}$

$= \sqrt[3]{(3^{2x})^3}$

$= 3^{2x}$ $\quad \because \sqrt[3]{a^3} = a$

(iii) $\frac{6(3)^{n+2}}{3^{n+1}-3^n}$ **09301136**

Solution

$\frac{6(3)^{n+2}}{3^{n+1}-3^n}$

$= \frac{6\left(3^n\times 3^2\right)}{3^n3^1-3^n\times 1}$ $\quad \left(\because a^{m+n} = a^m\times a^n\right)$

$= \frac{3^n\times 6\times 3^2}{3^n\left[3^1-1\right]}$

$= \frac{3^n\times 3^{-n}\times 6\times 9}{2}$

$$= \frac{3^{n-n} \times 6 \times 9}{2}$$

$$= \frac{3^0 \times 54}{2} \qquad (\because 3^0=1)$$

$= 1\times 27$

$= 27$

Q.8 The sum of three consecutive odd integers is 51. Find the three integers. 09301137

Solution

Sum = 51

Let x, $x+2$, $x+4$ be three consecutive odd numbers.

By condition:

$(x) + (x+2) + (x+4) = 51$

$x+ x+2+ x+4 = 51$

$3x+6 = 51$

$3x = 51-6$

$3x = 45$

$x = \frac{45}{3}$

$x = 15$

1st odd number = x = 15

2nd odd number = $x+2$

= 15+2 = 17

3rd odd number = $x+4$

= 15+4 = 19

Thus, 15, 17 and 19 are required three consecutive odd numbers.

Q.9 Abdullah picked up 96 balls and placed them into two buckets. One bucket has twenty-eight more balls than the other bucket. How many balls are in each bucket? 09301138

Solution

Total balls = 96

Let balls in 1st and 2nd bucket be x and y respectively.

By 1st condition:

x+y = 96------------ (i)

By 2nd condition:

$x = 28+y$

$x - y = 28$…… (ii)

Adding eq.(i) and (ii)

$$\begin{array}{l} x + \cancel{y} = 96 \\ x - \cancel{y} = 28 \\ \hline 2x \quad = 124 \end{array}$$

$x = \frac{124}{2} = 62$

Put it in eq. (i)

62 + y = 96

y = 96 −62

$\boxed{y = 34}$

Thus 1st bucket has 62 balls and 2nd bucket has 34 balls.

Q.10 Salma invested Rs. 3,50,000 in a bank which paid simple profit at a rate $7\frac{1}{4}\%$ per annum. After 2 years, the rate was increased to 8% per annum. Find the amount she had at the end of 7 years. 09301139

Solution

Time period = 7 years.

We divide the period of 7 years into two parts 2 years and 5 years.

Finding profit for 2 years

Principal amount = P = Rs.350,000/-

Profit rate = R = $7\frac{1}{4}\%$ or 7.25%

Time = t = 2 years

We know that

Profit = P×T×R

= 350,000×2×7.25%

$= 700,000 \times \frac{7.25}{100}$

$= 700,000 \times \frac{725}{100\times100}$

= 70 × 725

= Rs.50, 750

Finding the profit for 5 years

Principal amount = P = Rs.350,000/-

Profit rate = R = 8%

Time period = T = 5 years

We know that

Profit = P×T×R

= 350,000×5×8%

$= 1,750,0\cancel{0}\cancel{0} \frac{8}{1\cancel{00}}$

= Rs. 140,000/-

Finding the total profit:

Total profit = Rs. (50,750+140,000)

= Rs. 190,750

Finding the total amount:

Total amount at the end of 7 years

= Rs. (350,000 + 190,750)

= Rs. 540,750/-